Lateral Earth Pressure Theory

Prepared by: Prof. Rashid Mustafa
Assistant Professor & Head
Department of Civil Engineering
Katihar Engineering College, Katihar

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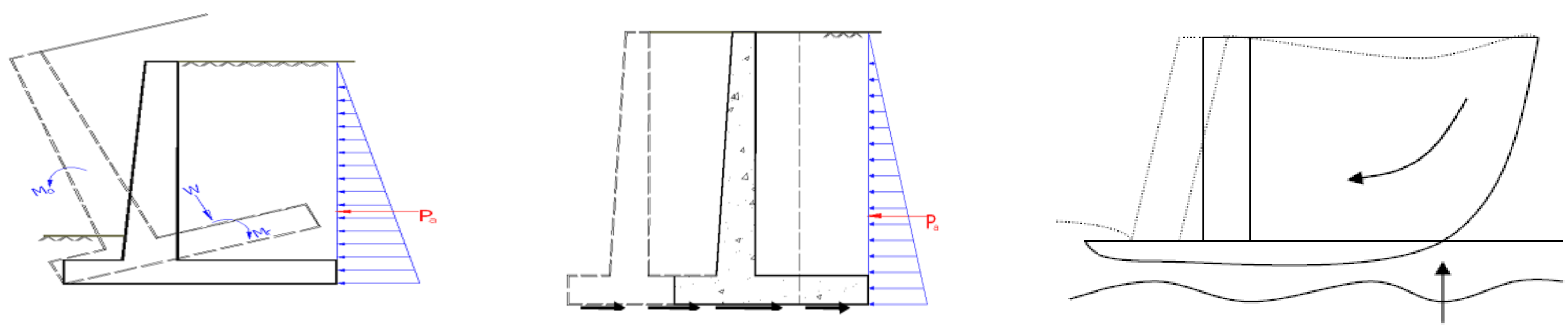
Introduction

- Retaining wall-Structures that are used to hold back earth and maintain a difference in the elevation in the ground surface.
- Retaining walls needs to withstand pressures from backfill, adjacent buildings, vehicular loads and dynamic events.
- Magnitude, distribution of earth pressure and lateral thrust are essential parameters in the designing and economize of retaining wall.
- Types of retaining wall failures :



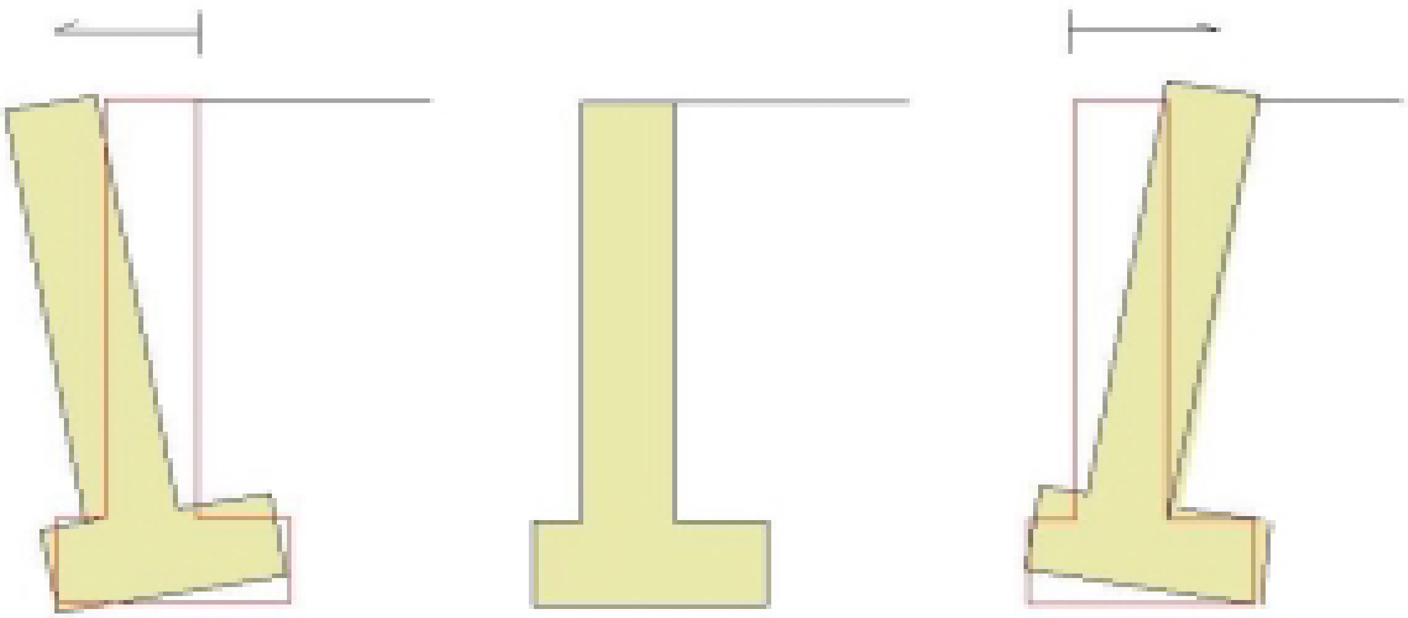
Typical Failure Modes of Retaining Wall

Lateral Earth Pressure is the function of:

- Type and amount of wall movement
- Shear strength parameter of soil
- Unit weight of soil
- Drainage condition in the backfill

Types of Earth Pressure

- At-Rest Earth Pressure
- Active Earth Pressure
- Passive Earth Pressure



Active Case
(Wall moves
away from soil)

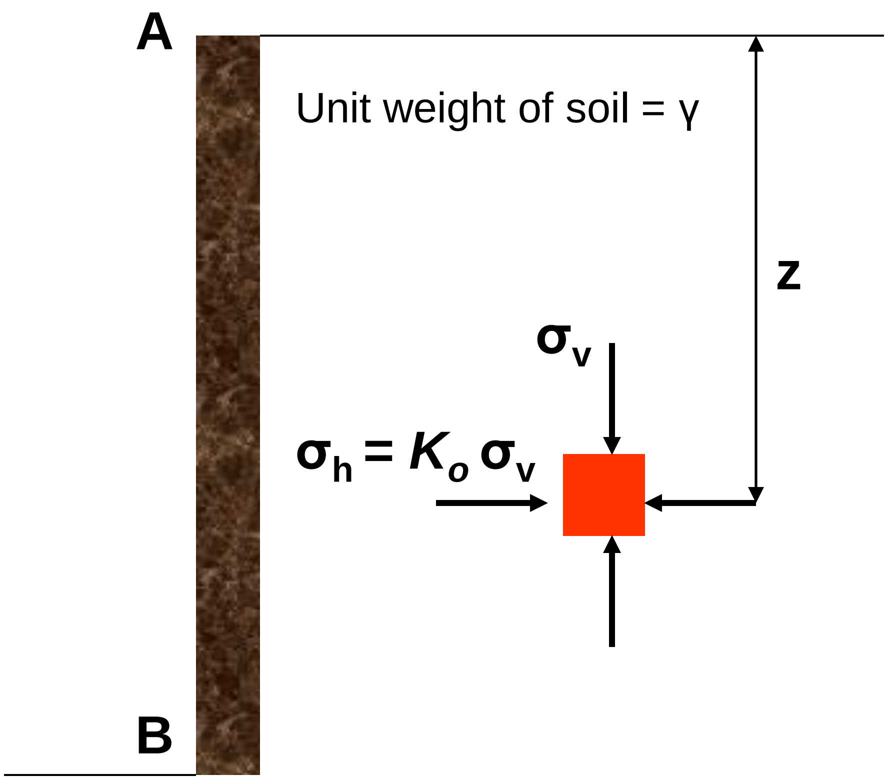
At Rest Case
(No movement)

Passive Case
(Wall moves
into soil)

Wall Movement

Source: www.pdhonline.org

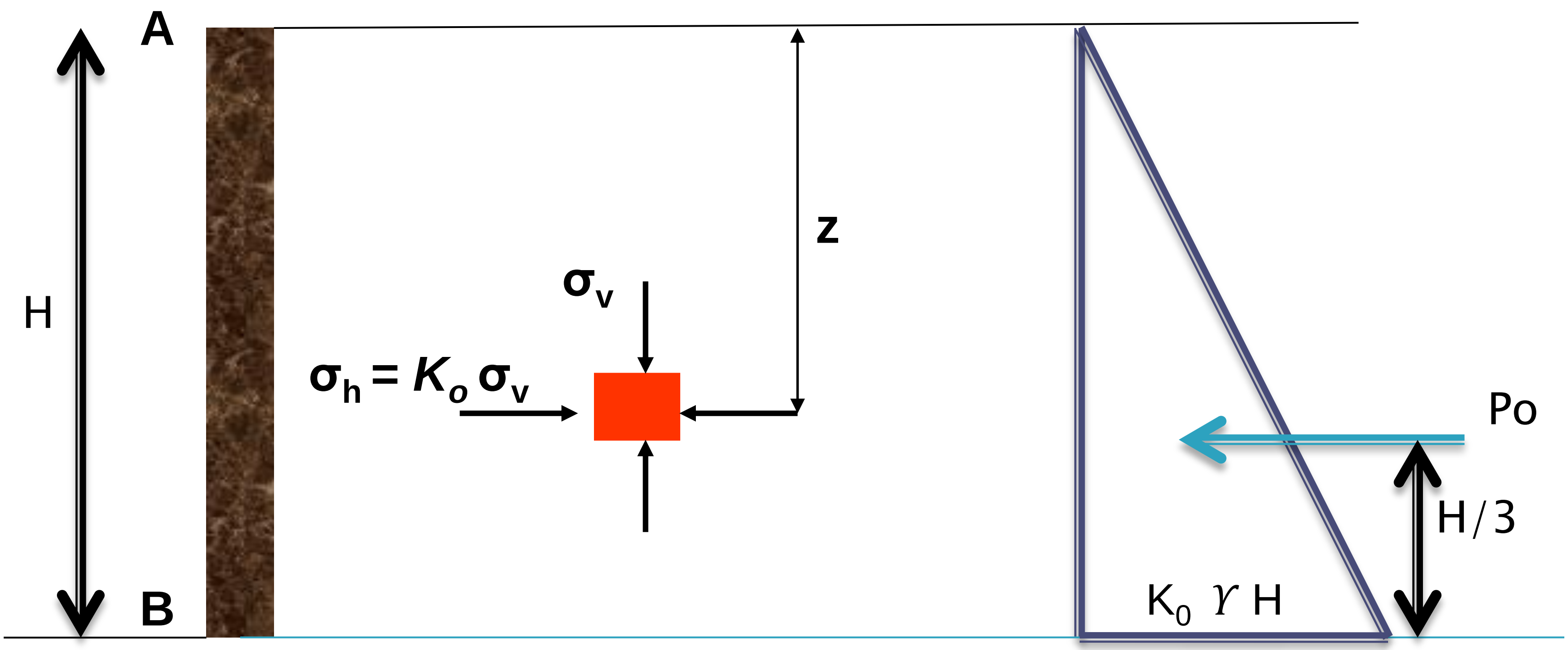
Earth Pressure At-rest



- If wall AB remains static soil mass will be in a state of elastic equilibrium horizontal strain is zero.
- Ratio of *horizontal stress* to *vertical stress* is called coefficient of earth pressure at rest, K_o

$$K_o = \frac{\sigma_h}{\sigma_v}$$

$$\sigma_h = K_o\, \sigma_v = K_o \gamma\, z$$



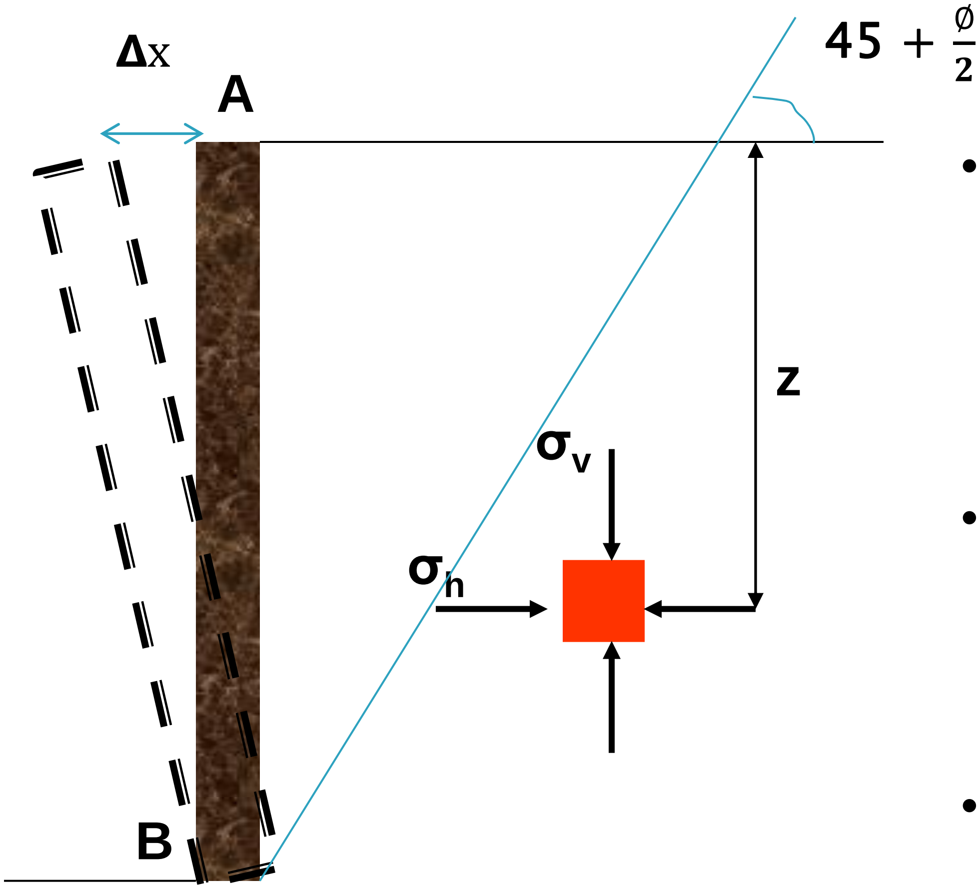
Active pressure at rest $p_0 = K_0 \cdot \Upsilon \cdot H$

Where , K_0 = Coefficient of Earth Pressure at rest

- For Normally Consolidated Clay ,
  $K_0 = 1 - \sin \Phi$ (Jacky ,1944)

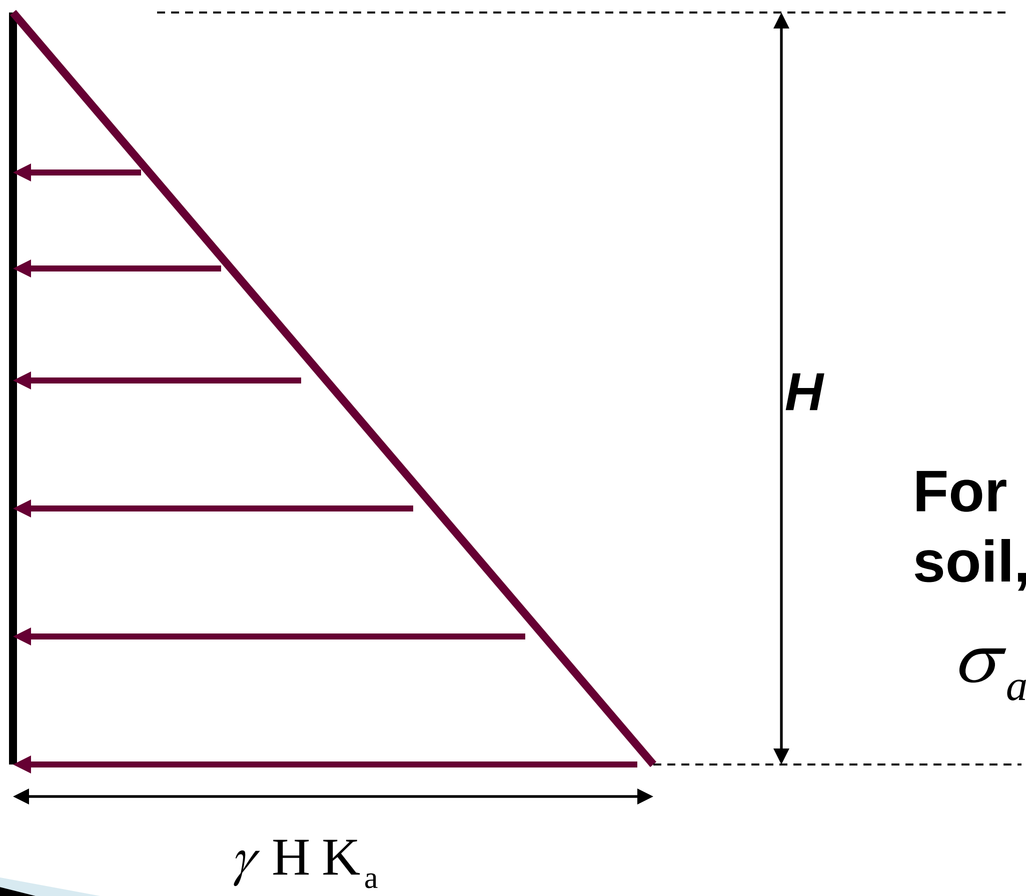
- For an over consolidated clay
  $(K_0)_{\text{Over consolidated}} = (K_0)_{\text{Normally consolidated}} \cdot (OCR)^{0.5}$

Active Earth Pressure



- Plastic equilibrium in soil refers to the condition where every point in a soil mass is on the verge of failure.
- If wall AB is allowed to move away from the soil mass gradually, *horizontal stress* will decrease.
- This is represented by Mohr's circle in the subsequent slide.

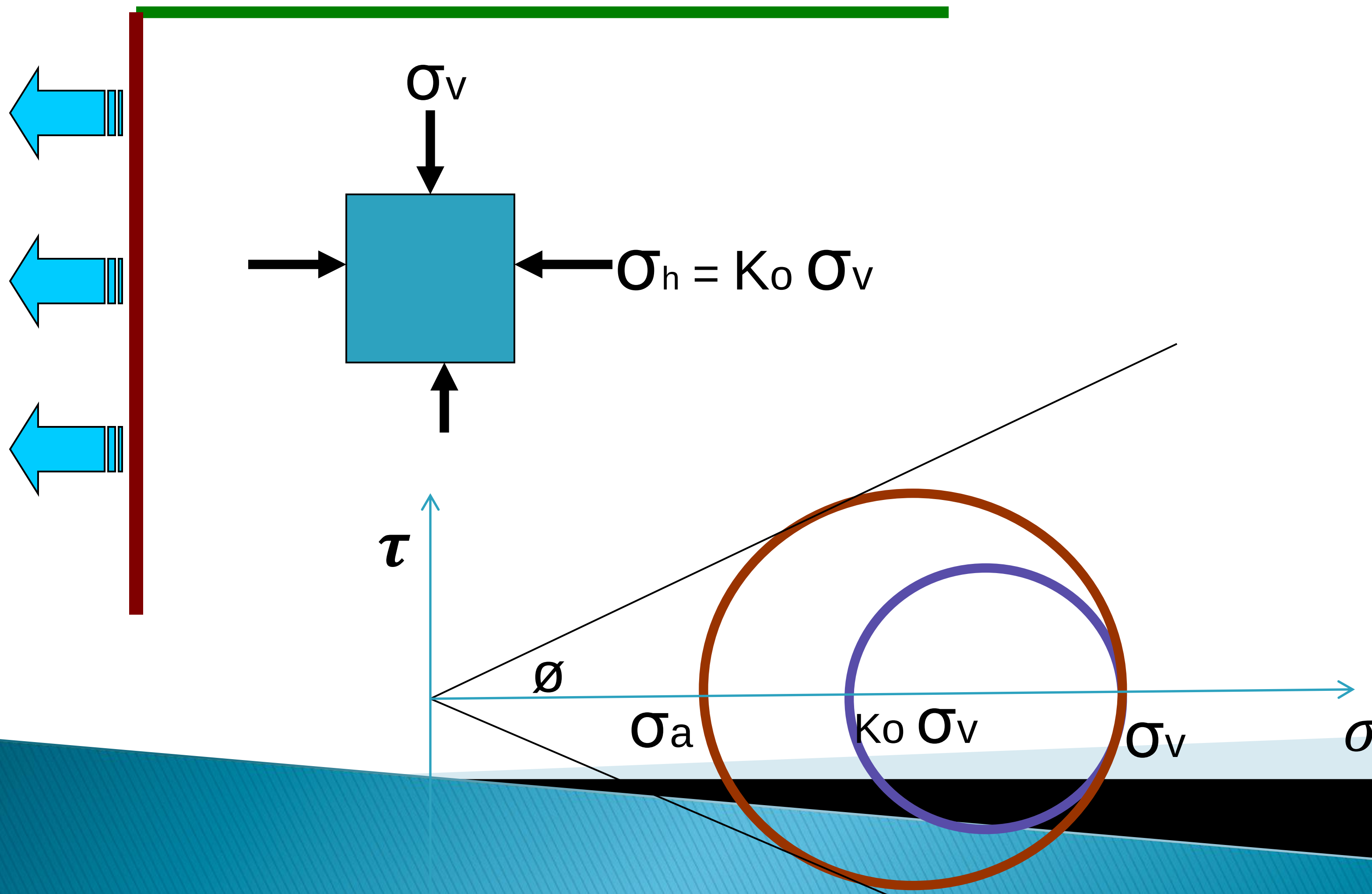
Active pressure distribution for *Φ* soil



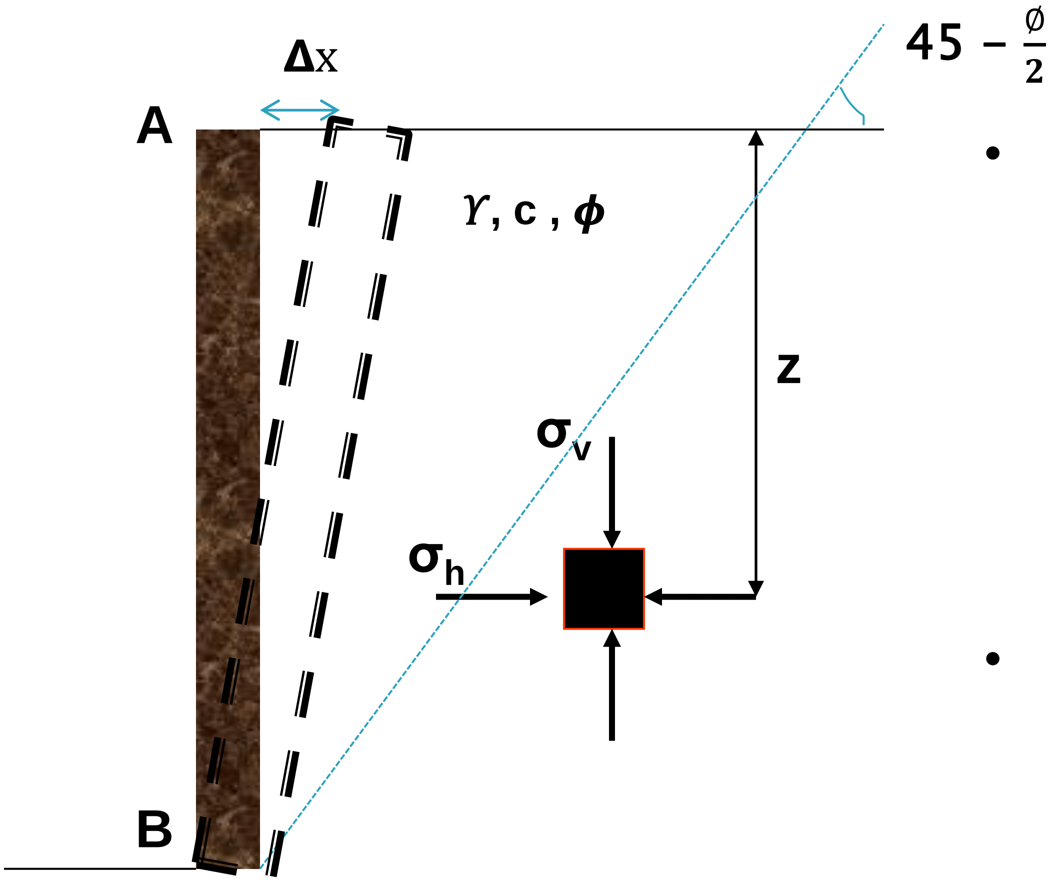
For cohesion-less soil, c = 0

$$\sigma_a = \sigma_v \, K_a = \gamma \, H \, K_a$$

Mohr Circle for Active Condition

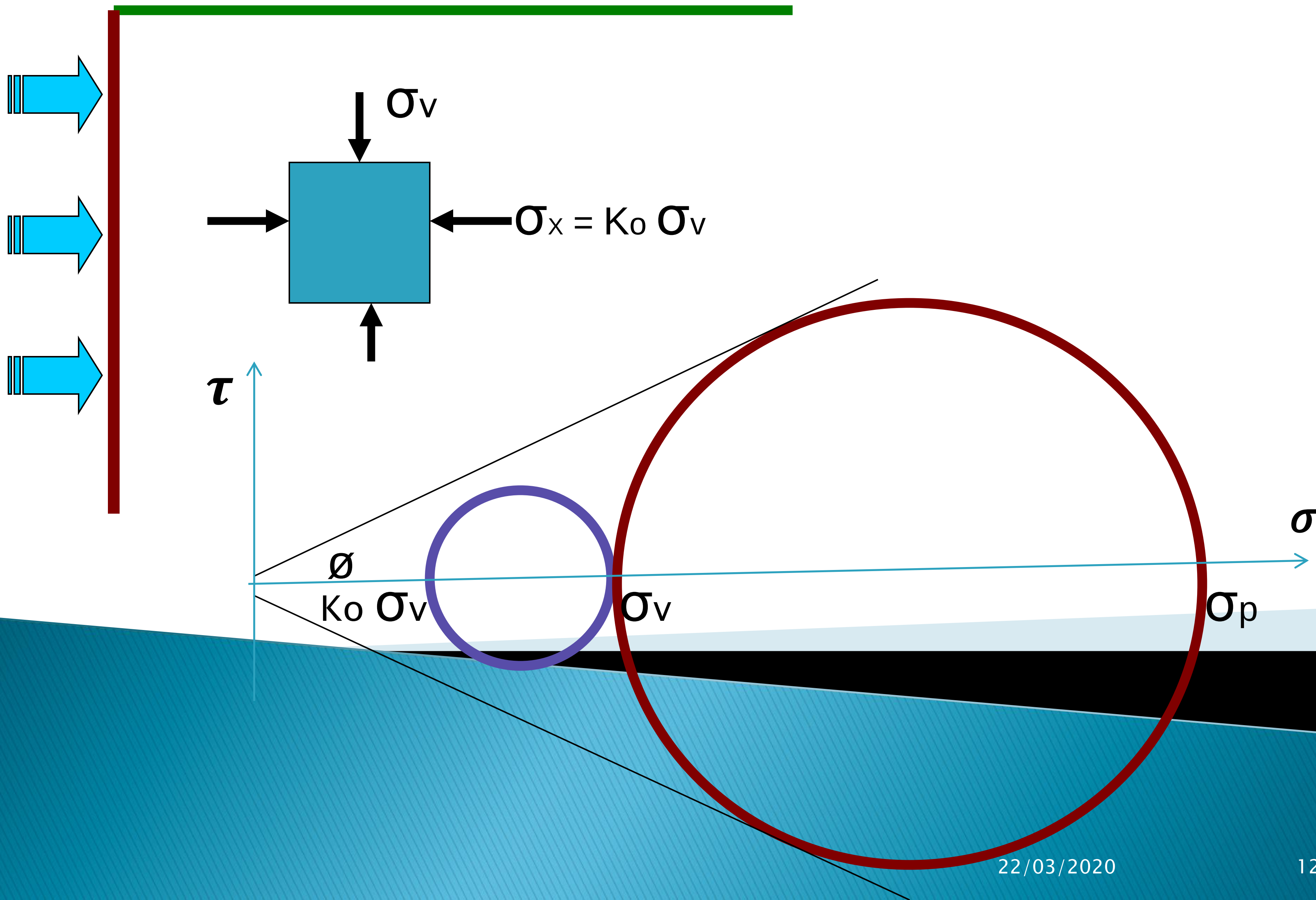


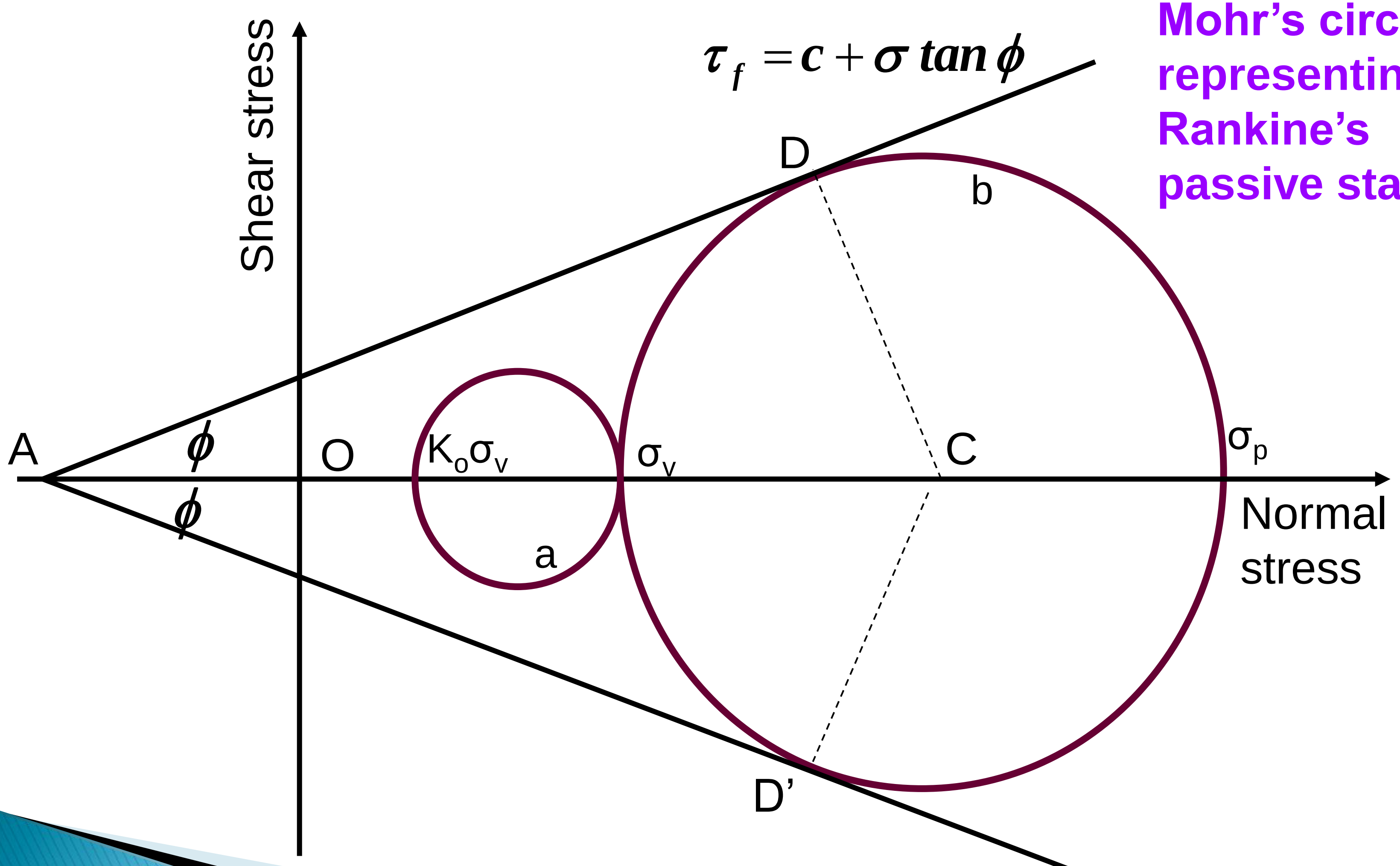
Passive Earth Pressure



- If the wall is pushed into the soil mass, the principal stress σ_h will increase. On the verge of failure the stress condition on the soil element can be expressed by Mohr's circle b.
- The lateral earth pressure, σ_p, which is the major principal stress, is called Rankine's passive earth pressure

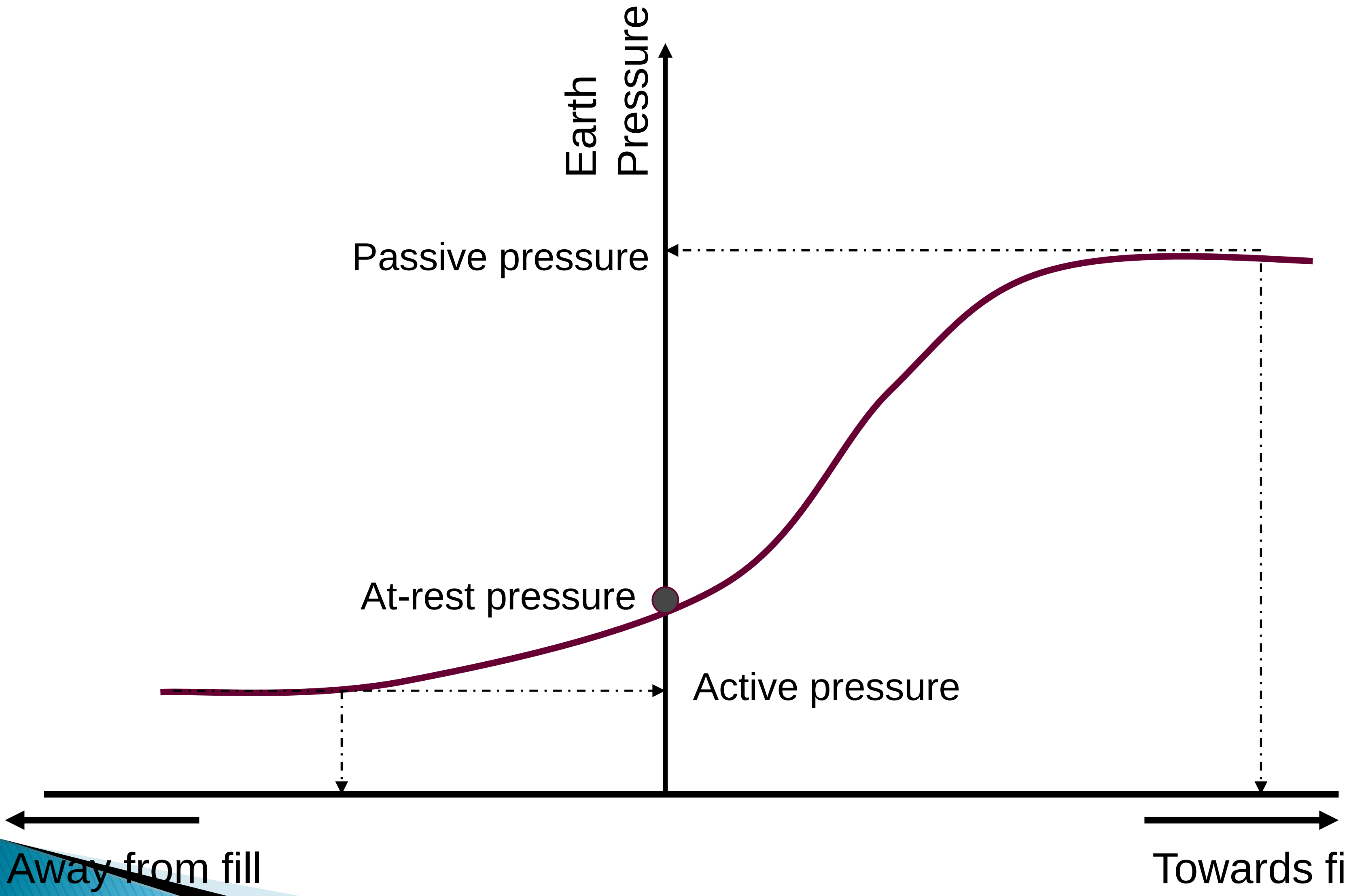
Mohr Circle for Passive Condition





Mohr's circle representing Rankine's passive state.

Variation of Earth pressure with the movement of wall



LATERAL EARTH PRESSURE THEORY

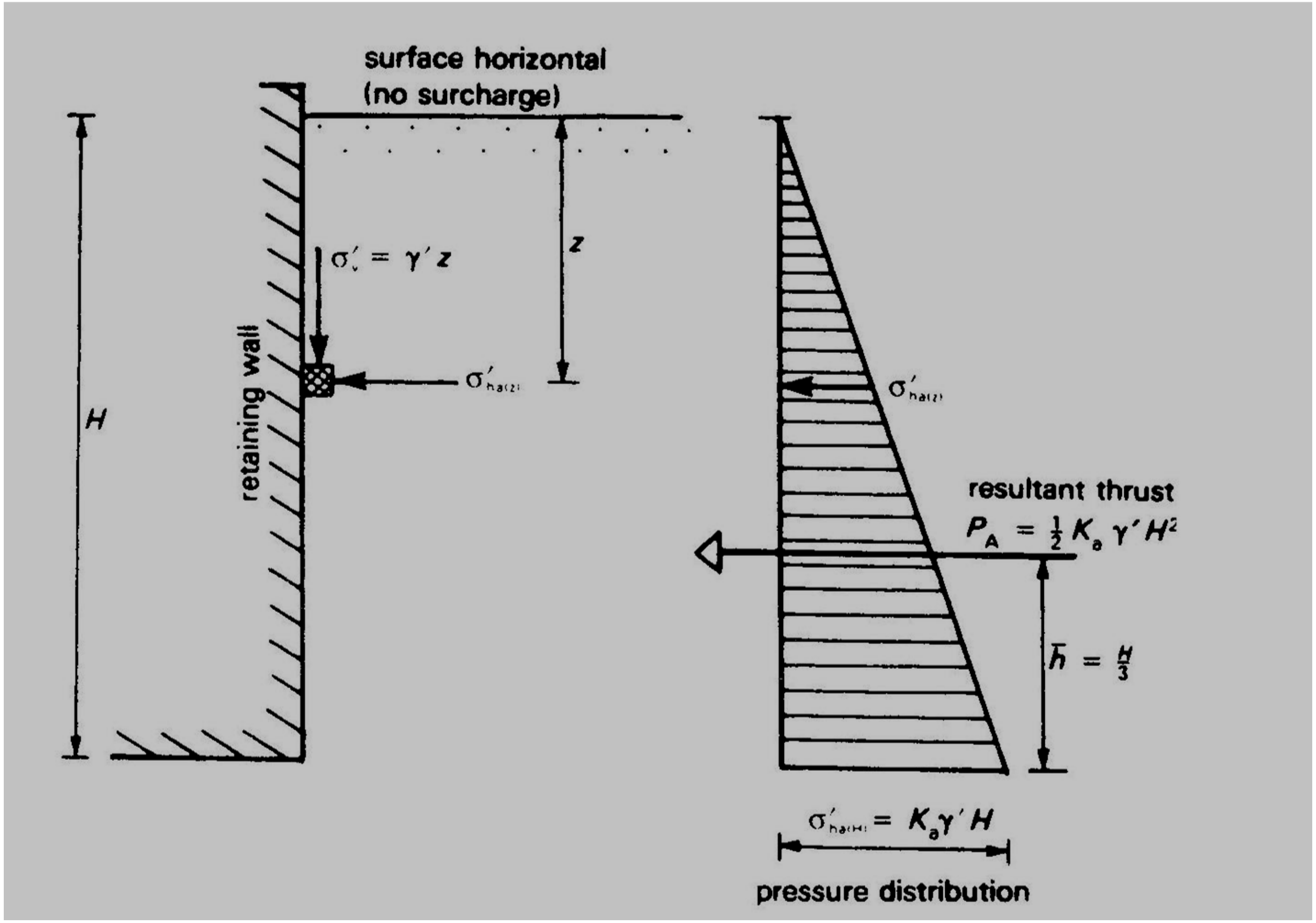
Rankine's Theory (1857)

- **Develop based on semi infinite "loose granular" soil mass for which the soil movement is uniform.**
- **Used stress states of soil mass to determine lateral pressures on a frictionless wall**

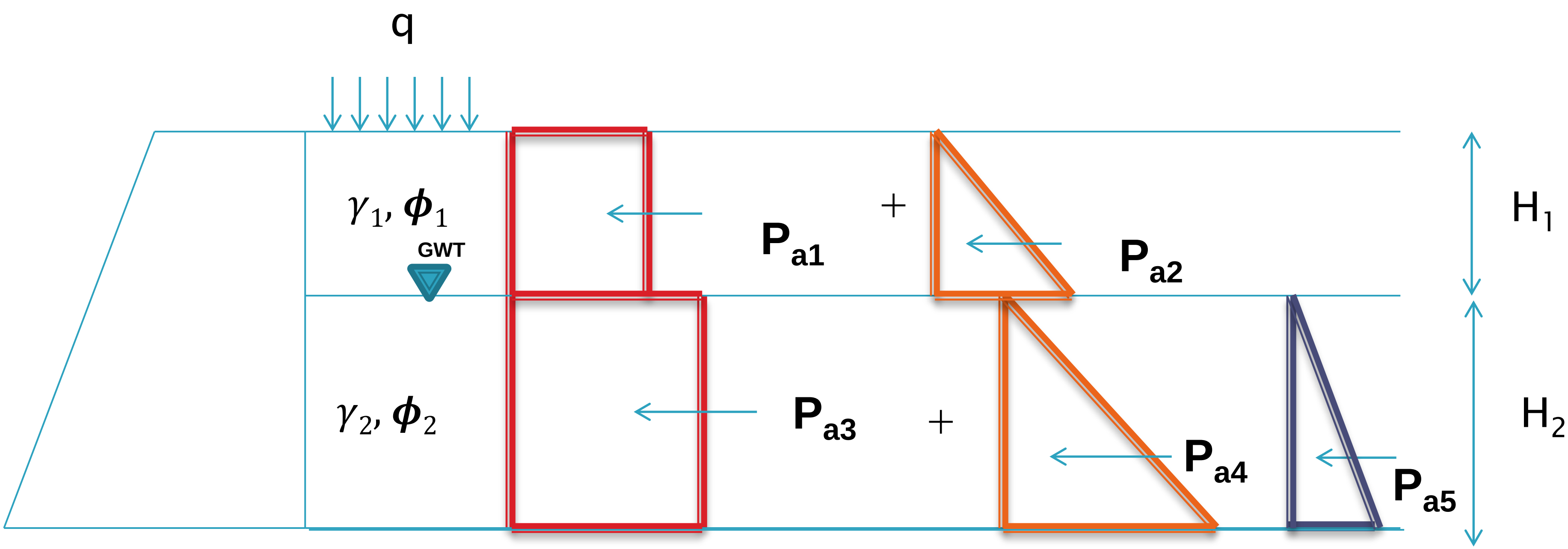
Assumptions :

- **Vertical frictionless wall**
- **Dry homogeneous soil**
- **Horizontal surface**

Active Earth Pressure for Cohesion-less Soil



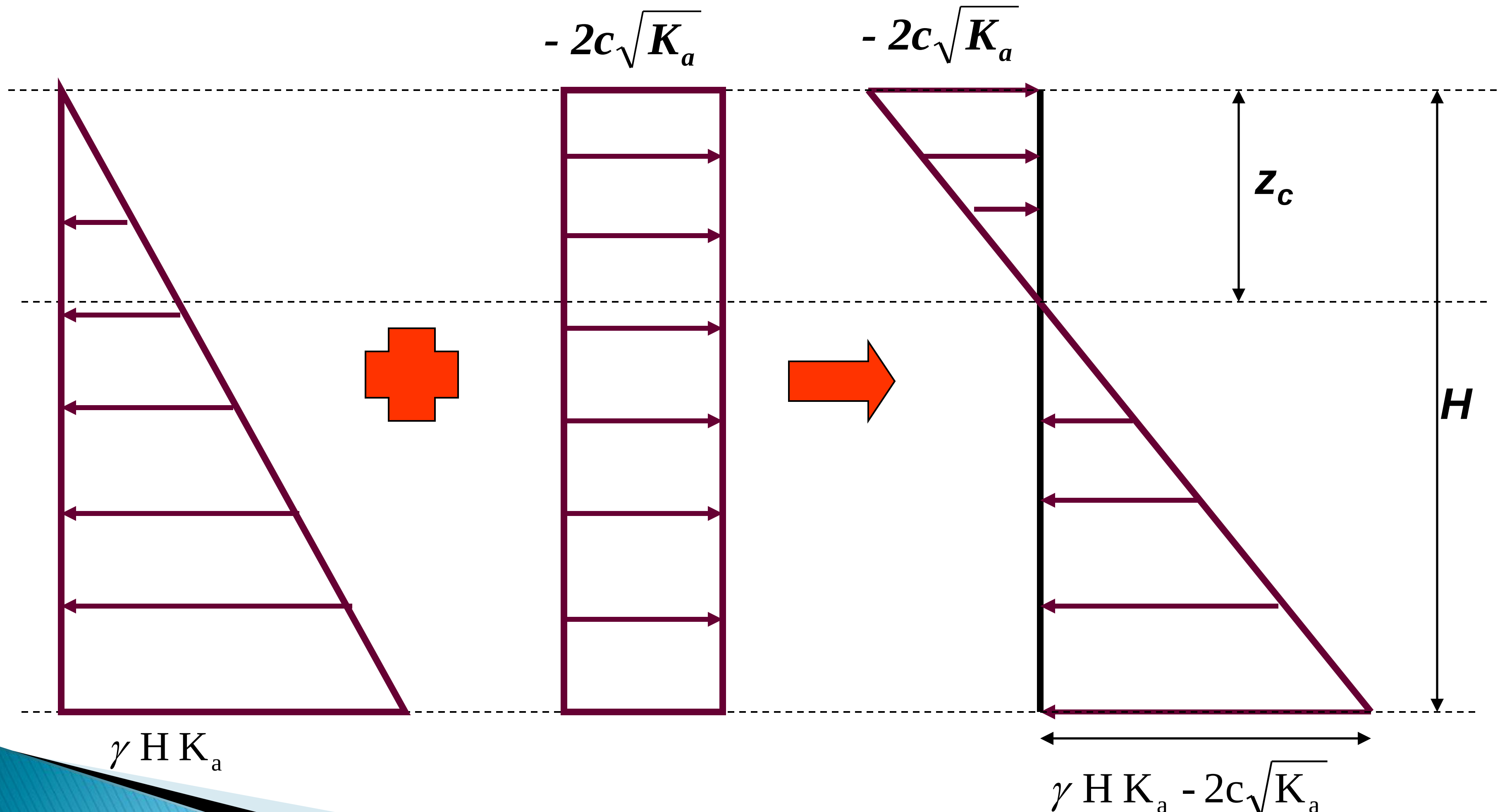
Effect of surcharge in stratified soil



Total Rankine active force per unit length of wall = $P_{a1} + P_{a2} + P_{a3} + P_{a4} + P_{a5}$

$$\bar{x} = \frac{P_{a1} \times \left(H_2 + \frac{H1}{2}\right) + P_{a2} \times \left(H_2 + \frac{H1}{3}\right) + P_{a3} \times \left(\frac{H2}{2}\right) + P_{a4} \times \left(\frac{H2}{3}\right) + P_{a5} \times \left(\frac{H2}{3}\right)}{P_a}$$

Active pressure distribution for c - ϕ soil (Resal and Bell theory)



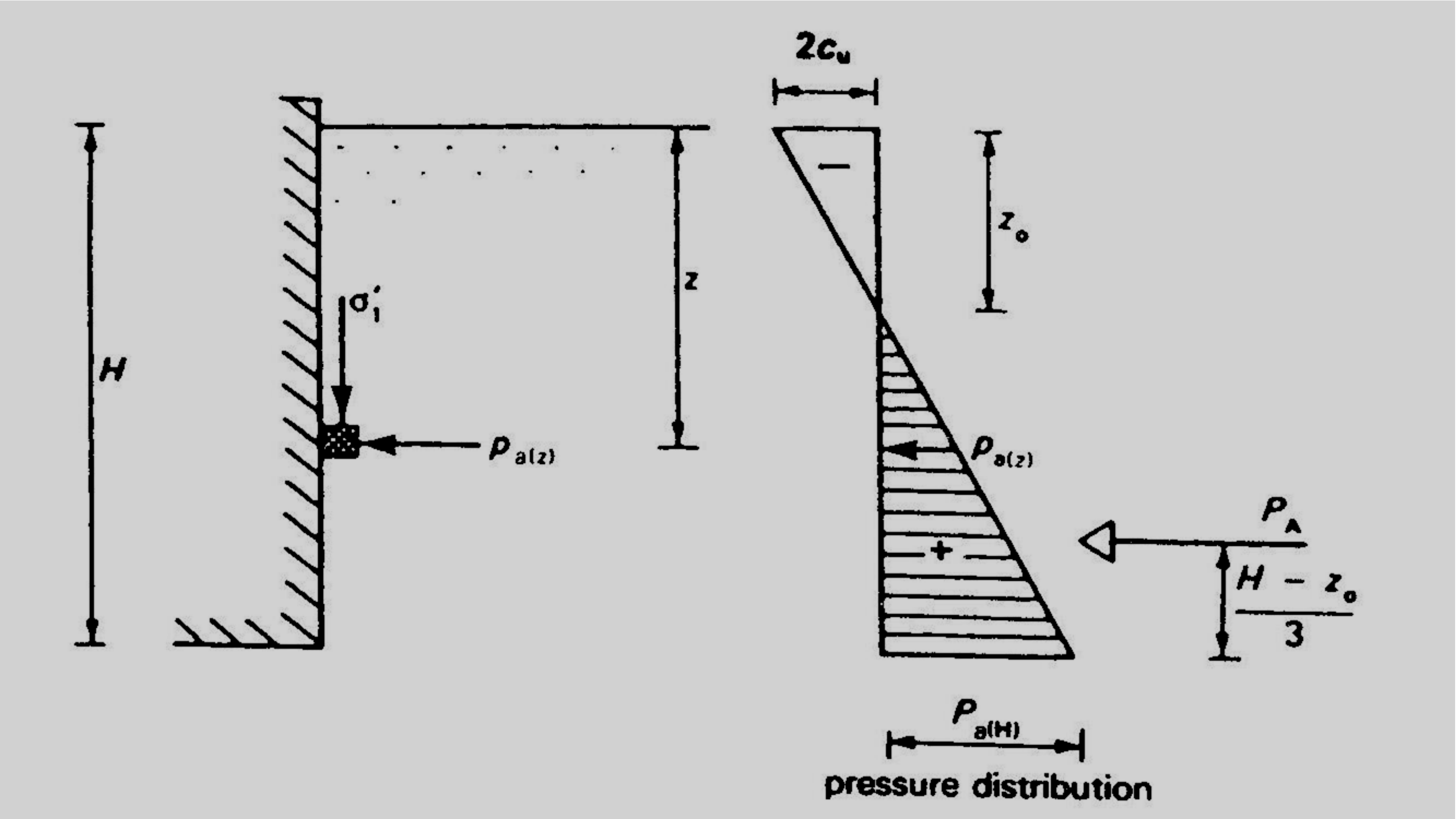
$$\sigma_a = \gamma \, H \tan^2(45 - \frac{\phi}{2}) - 2c \tan(45 - \frac{\phi}{2})$$

$$= \gamma \, H \, K_a - 2c\sqrt{K_a}$$

$$Ratio \ \frac{\sigma_a}{\sigma_v} = coefficient \ of \ Rankine's \ active \ earth \ pressure$$

$$K_a = \frac{\sigma_a}{\sigma_v} = tan^2(45 - \frac{\phi}{2}) = \frac{1 - \sin\phi}{1 + \sin\phi}$$

Tension cracks in cohesive soils



Depth of Tensile crack(Z_o)

$$Zc = \frac{2c}{\gamma\sqrt{K_a}}$$

Where Z_c is depth of tension crack

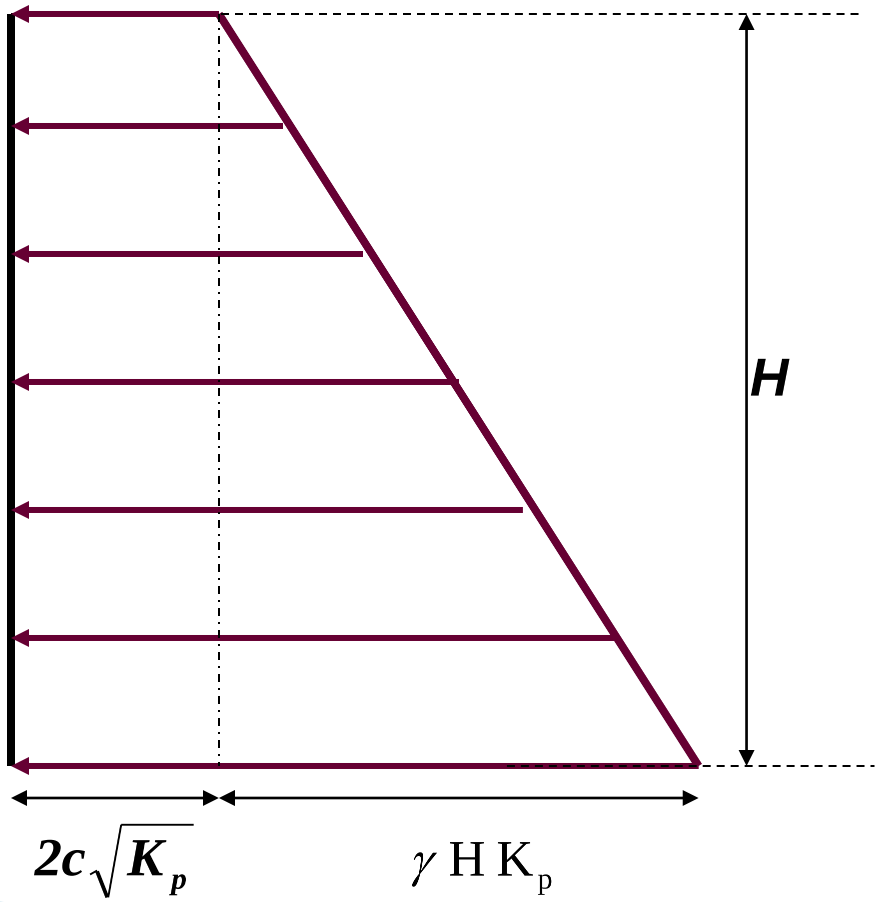
- For pure cohesive soil, i.e. when $\phi = 0$

$$zc = \frac{2c}{\gamma}$$

Critical depth of vertical cut (H_c)

$$H_c = 2\,Z_c = \frac{4c}{\gamma\sqrt{K_a}}$$

Passive Earth Pressure distribution for c- ϕ Soil



For cohesion less soil,

$$\sigma_p = \sigma_v K_p = \gamma H K_p$$

Computation of Active Thrust before and after the tension crack

- Total Active thrust before tension crack(P_a) = $\int_0^H p_a.\mathrm{dz}$

Where,

$$p_a = (k_a.\Upsilon.\mathrm{z} - 2\mathrm{c}.\sqrt{ka})$$

- Total active thrust after the tension crack (P_a) = $\int_{zc}^{H} p_a.dz$

Where,

z_c is the depth of tension crack

p_a is the active earth pressure at any depth z

H is the height of retaining wall

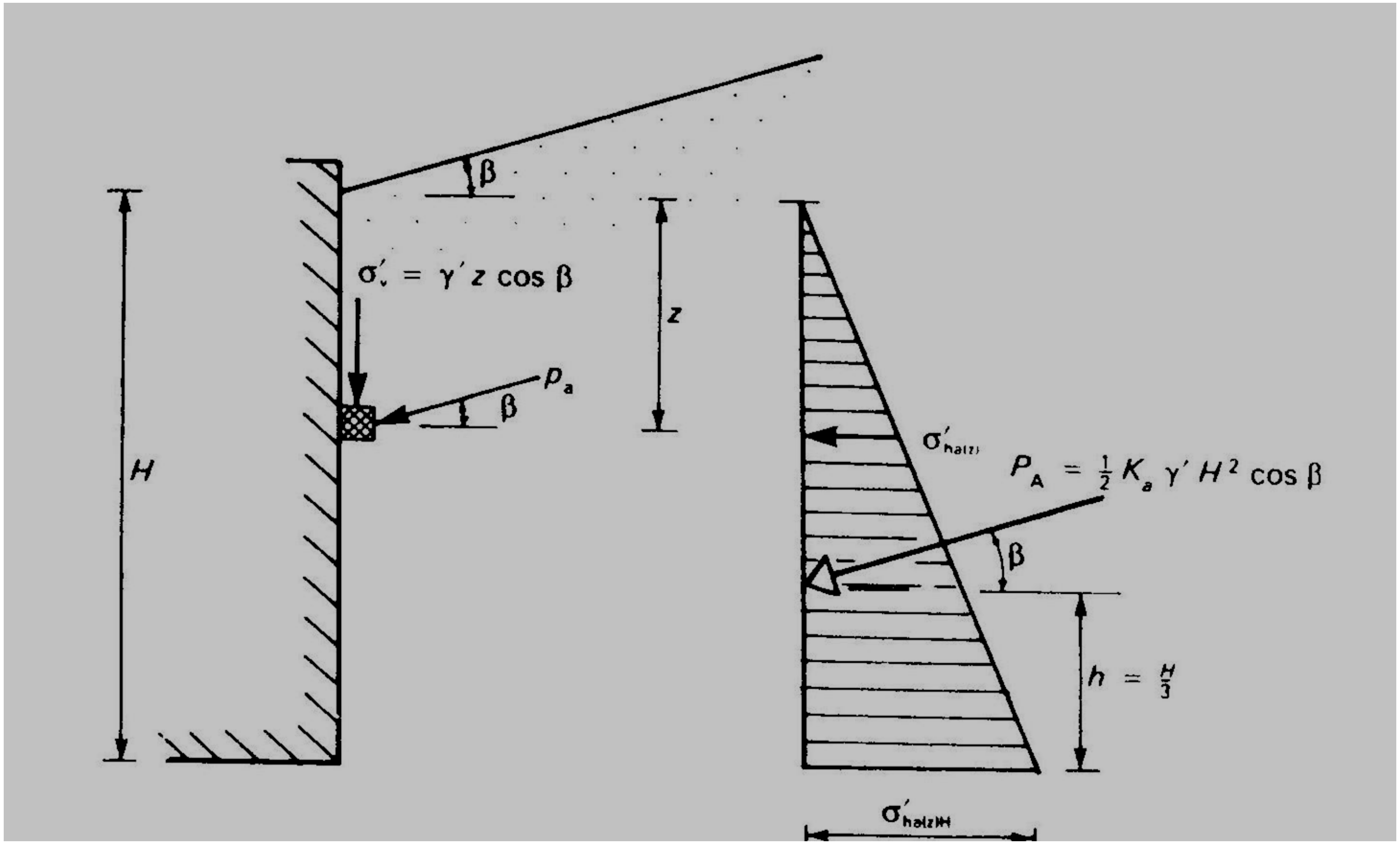
$$\sigma_p = \sigma_v \tan^2(45 + \frac{\phi}{2}) + 2c \tan(45 + \frac{\phi}{2})$$

$$= \gamma H K_p + 2c\sqrt{K_p}$$

- **For cohesion-less soil**

$$\frac{\sigma_p}{\sigma_v} = K_p = \tan^2(45 + \frac{\phi}{2}) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

Effect of Sloping Surface



Active pressure,

$$p_a = K_a\ \sigma_v$$

Passive pressure,

$$p_p = K_p\ \sigma_v$$

where

$$K_a = Cos\beta\ \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi')}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi')}}$$

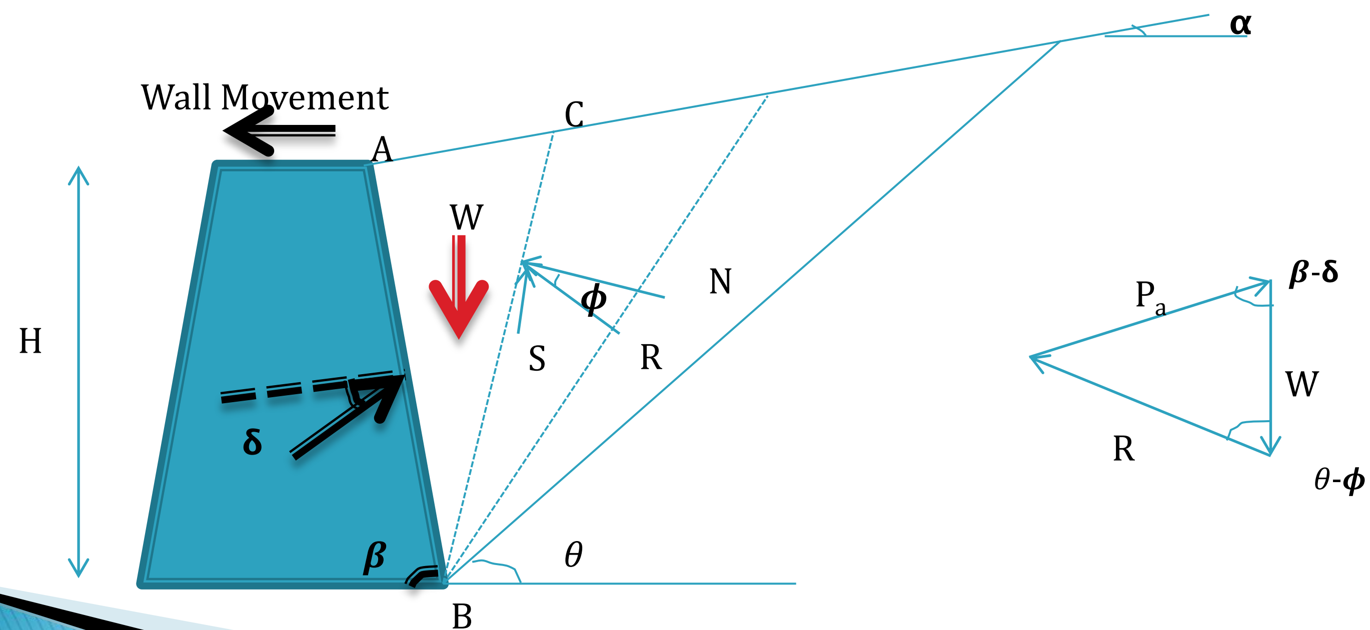
and

$$K_p = Cos\beta\ \frac{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\phi')}}{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\phi')}} = \frac{1}{K_a}$$

Coulomb's Wedge Theory (1776)

Assumptions

- Backfill is dry ,cohesion less ,homogeneous ,isotropic and ideally plastic material.
- The slip surface is plane which passes through the heel of the wall.
- The sliding wedge its self acts as rigid body .
- Wall friction taken into accounts.



$$K_a = \frac{\text{Sin}^2(\beta+\phi)}{\text{Sin}^2\beta.\text{Sin}(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\alpha)}{\sin(\beta-\delta).\sin(\alpha+\beta)}}\right]^2}$$

- In actual design of retaining walls , angle of wall friction **δ** is assume to be between $\boldsymbol{\phi}/2$ to $2\boldsymbol{\phi}/3$.

Applying sine law in force triangle,

$$\frac{\boldsymbol{P_a}}{\boldsymbol{\theta-\phi}} = \frac{\boldsymbol{W}}{\boldsymbol{Sin}[\mathbf{180}-(\boldsymbol{\theta}-\emptyset+\boldsymbol{\beta}-\boldsymbol{\delta})}$$

Important Links

- https://nptel.ac.in/courses/105/105/105105168/
- https://nptel.ac.in/courses/105/104/105104162/
- https://nptel.ac.in/courses/105/104/105104147/
- https://www.youtube.com/watch?v=ucbinKVZvF8
- https://nptel.ac.in/courses/105/105/105105176/

References

- Murthy, V.N.S. “ Principles and practices of soil engineering and foundation engineering”
- Murthy, V.N.S. “ Principles and practices of soil engineering and foundation engineering”
- Ranjan, Gopal and Rao A.S.R. “Basic and applied soil mechanics”
- www.wikipedia.com
- www.pdhonline.org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