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Subject : Soil and Rock Mechanics
Level : B. Tech VI th Semester
Topic : Shear strength of soil

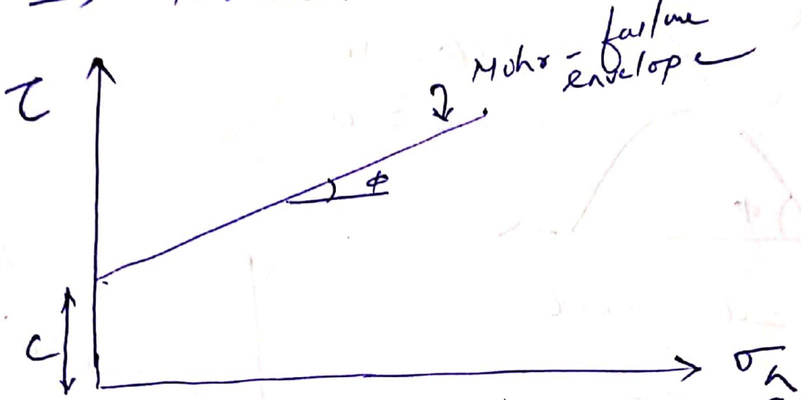
SHEAR STRENGTH OF SOIL

* Shear strength is the capacity to resist shear stress.
* Shear strength of soil governs bearing capacity of soil, stability of slopes, Earth retaining structure.
* Shear strength is given by

$$\tau = c + \sigma_n \tan\phi$$

$c \rightarrow$ Cohesion, $\phi \rightarrow$ Angle of internal friction

$\sigma_n \rightarrow$ Normal stress



The above theory is not valid for the condition where W/T is present. Hence the above theory is modified & improved by Terzaghi which is called modified Mohr-Coulomb theory.

$$\tau = c' + \sigma_n' \tan\phi'$$

$c' \rightarrow$ Effective cohesion

$\phi' \rightarrow$ Effective friction angle

Graphical representation of Coulomb Eqn.

$\sigma_n' = \sigma_n - U =$ Effective normal stress on critical plane

c & ϕ are known as shear strength parameter

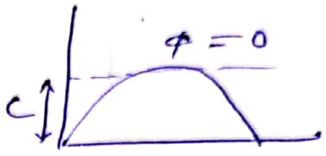
→ c & ϕ are not the inherent properties of soil. These are the related to the type of test & the conditions under which these are measured.

→ c, ϕ → Total stress parameter

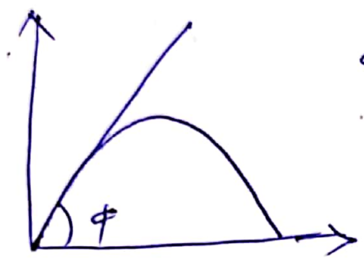
→ c', ϕ' → Effective stress parameter.

→ For Pure Clay (c – soil)

$c = \kappa$ $\quad$ $\phi = 0$



→ For Pure SAND (ϕ – soil)



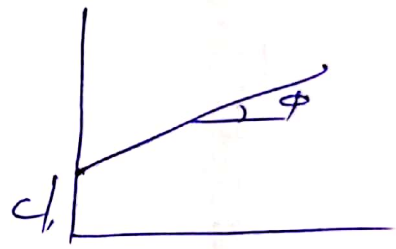
$\phi = \kappa$

$c = 0$

→ For $c - \phi$ soil

$c = \kappa$

$\phi = \kappa$



⇒ For Clay

$$\tau = c$$

For SAND

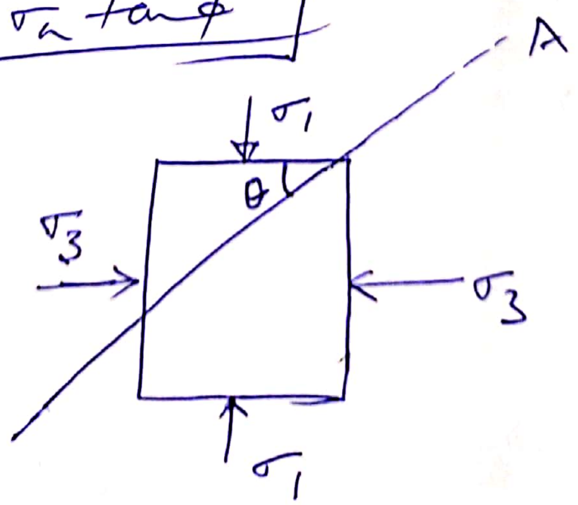
$$\tau = \sigma_n \tan\phi$$

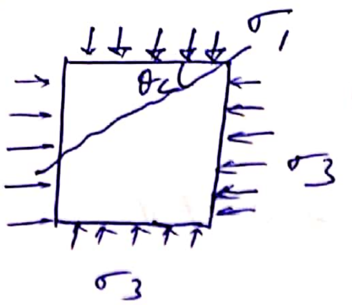
⇒ For $c - \phi$ soil

$$\tau = c + \sigma_n \tan\phi$$

⇒ 2D Analysis

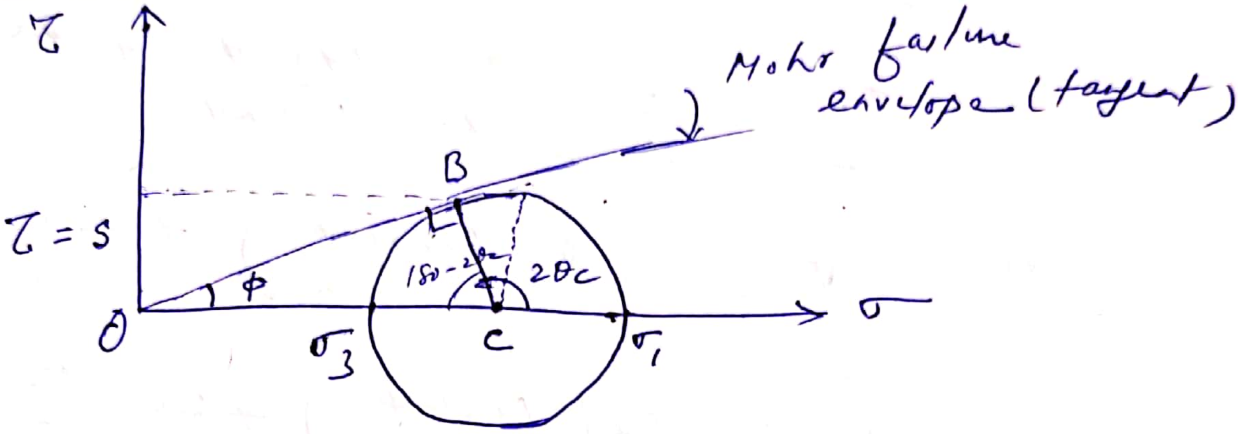
Consider a plane A–A at an angle θ with major principle plane.





$$\theta_c = 45 + \frac{\phi}{2}$$

⇒ Mohr – Circle for SAND



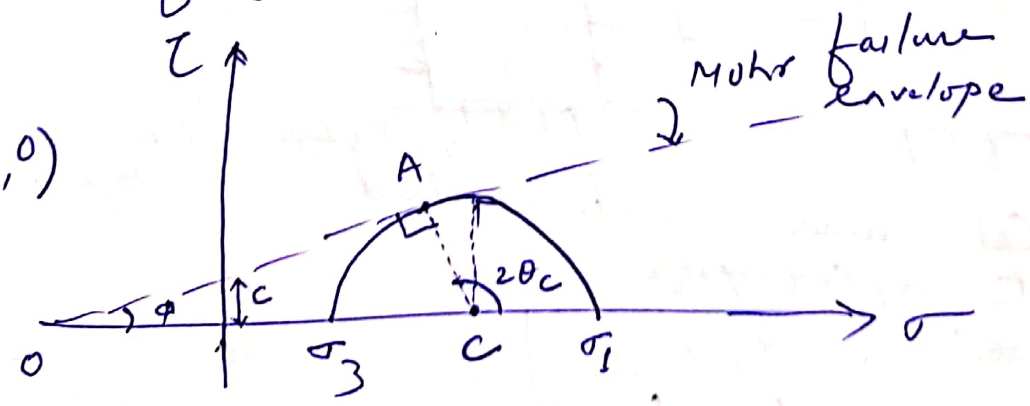
$$\phi + 90° + 180 - 2\theta_c = 180°$$

$$\theta_c = 45 + \frac{\phi}{2}$$

→ failure plane makes an angle of $(45 + \frac{\phi}{2})$ degree with the major principal plane.

⇒ Relation b/w major & minor principal ~~plan~~ stress at failure in a soil mass on the basis of Mohr – Coulomb criteria of failure.

$$c = \left(\frac{\sigma_1 + \sigma_3}{2}, 0\right)$$



$$\sin\phi = \frac{\left(\frac{\sigma_1 - \sigma_3}{2}\right)}{\left(\frac{\sigma_1 + \sigma_3}{2}\right) + c\cot\phi}$$

$$\left(\frac{\sigma_1 + \sigma_3}{2}\right)\sin\phi + c\cos\phi = \frac{\sigma_1 - \sigma_3}{2}$$

$$\frac{\sigma_1}{2}(1 - \sin\phi) = \frac{\sigma_3}{2}(1 + \sin\phi) + c\cos\phi$$

$$\frac{\sigma_1}{2} = \frac{\sigma_3}{2}\,\frac{1+\sin\phi}{1-\sin\phi} + \frac{2c\cos\phi}{1-\sin\phi}$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + \frac{2c\cos\phi}{1-\sin\phi}$$

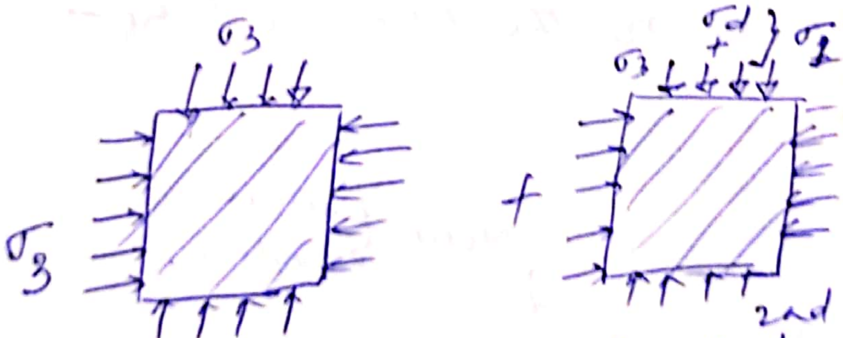
$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1-\sin^2\phi}{(1-\sin\phi)^2}}$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\boxed{\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right) + 2c\tan\left(45 + \frac{\phi}{2}\right)}$$

Similarly,

$$\sigma_3 = \sigma_1\left(\frac{1-\sin\phi}{1+\sin\phi}\right) - 2c\sqrt{\frac{1-\sin\phi}{1+\sin\phi}}$$



Cell Pressure
1st stage / Confining stage

2nd stage / Deviator stage / Shear stage
deviator load

$$\boxed{\sigma_1 = \sigma_3 + \sigma_d}$$

σ_3 → Cell pressure. σ_d → Deviator stress

→ Eqn No (1) can also be written in effective stress parameter.

$$\sigma_1' = \sigma_3' \tan^2(45 + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2})$$

$$\sigma_1' = \sigma_1 - U$$

$$\sigma_3' = \sigma_3 - U$$

Test on shear strength of soil

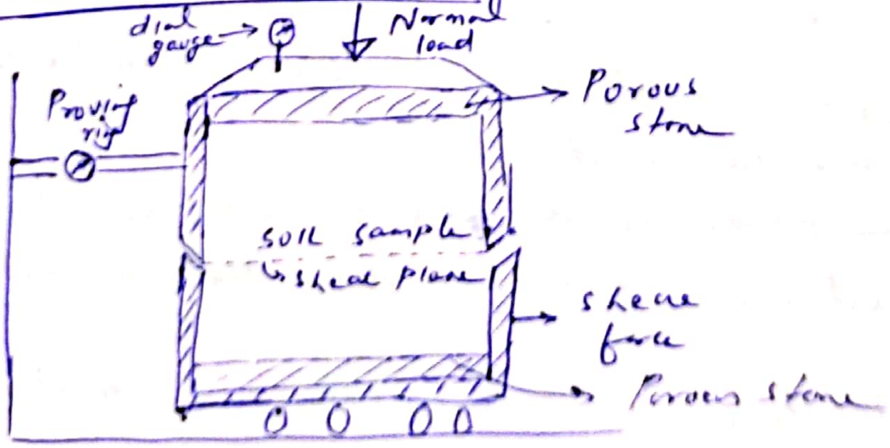
Lab Test

1. Direct shear test or Box shear test (SANDY SOIL)
2. Triaxial shear test (All types of soil)
3. Vane shear test (soft clay)
4. Unconfined Compression test (CLAY)

Field Test

1. Vane shear test
2. Standard Penetration test (SPT)
3. Cone penetration test (CPT)

1. Direct shear test



→ Direct shear test is ~~used for~~ also called ~~Shear box~~ box shear test

→ Test is done on shear box

→ Shear box is either square or circular shape

→ Box dimension (square) → 60x60 mm

→ Vertical load is applied through loading plate

→ Soil is sheared gradually by applying horizontal force.

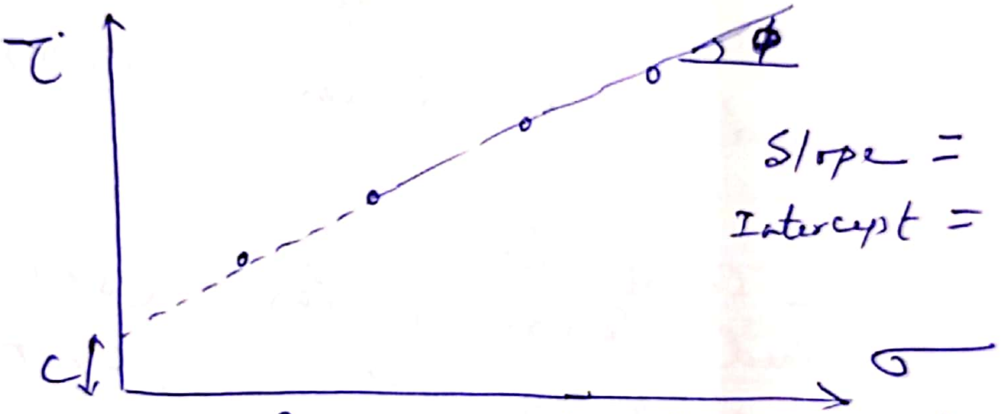
→ Magnitude of shear load is measured by Proving ring.

→ ~~Shear~~ Normal stress = Normal load / Nominal area of specimen

→ Shear stress (τ) $= \frac{F}{A} = \frac{K \cdot N_1}{A}$

K → Proving ring Constant (N/μm)

→ Effective stress & Total stress are same in this test



Slope = φ

Intercept = c

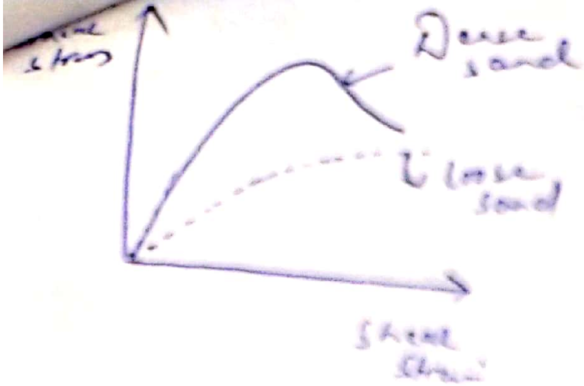
Advantage: (i) Quick, simple & inexpensive

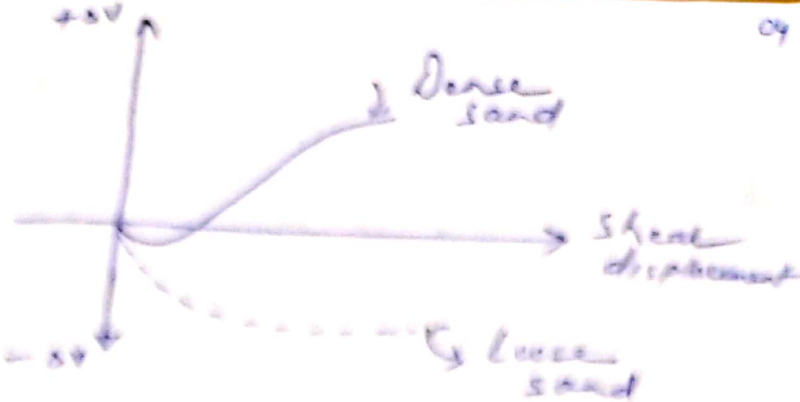
(ii) Easy to prepare sample.

~~(iii)~~ Disadv:

1. Drainage cond^n can't be controlled & PWP can't be measured.
2. Failure plane is always horizontal & ~~Predetermi~~ Predetermined which may not be the weakest plane.

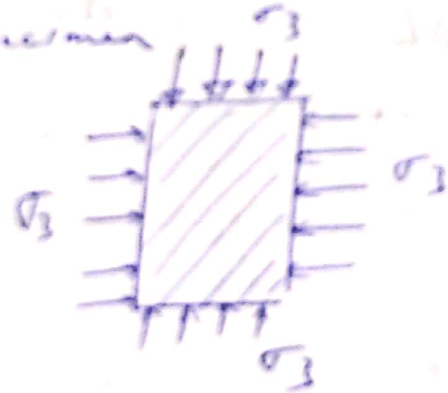
→ At least three tests are required to draw a Line properly.





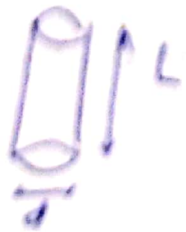
⇒ Triaxial test :

- This is the most widely used shear strength test & is suitable for all types of soil
- Drainage can be controlled, what ever be the type of soil.
- PWP can be measured.
- Failure plane is not pre determined
- This test is performed in two stages.
- The 1st stage is called confining stage in which hydrostatic pressure or water pressure acting on the specimen



1st stage / confining stage / Cell Pressure stage

→ Specimen is in the shape of cylinder.

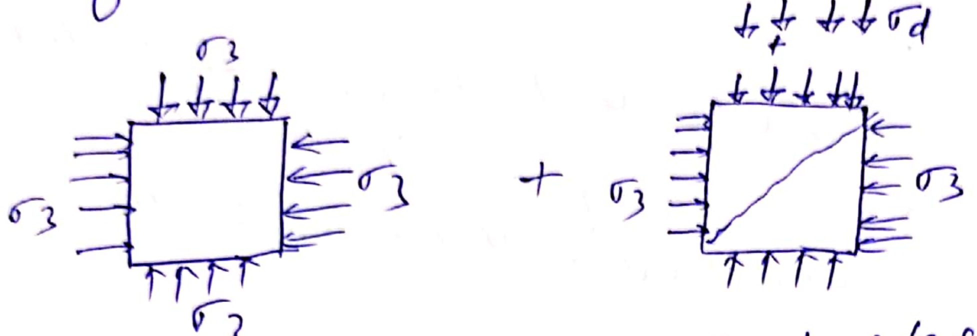


$\frac{L}{d} = 2$

→ Size of specimen: 76 × 38 mm ~~38×76~~ mm or 100mm × 50mm

→ The triaxial cell is filled with water & specimen is sealed inside a rubber membrane. Cell pressure is applied (called confining pressure σ_3)

→ With cell pressure held constant, additional axial stress is applied gradually until sample fails at additional axial stress value of σ_d



1st stage or Confining stage or Cell pressure stage

2nd stage or deviator stage

→ $\sigma_1 = \sigma_3 + \sigma_d$ ⇒ $\sigma_d = \sigma_1 - \sigma_3 = \frac{\text{Axial load}}{A_c}$

→ A_c → Corrected Area

→ $V = A(L \pm \Delta L) = V_0 \pm \Delta V$

A_0 → Original Area

A_c → Corrected Area

~~$A_c = \frac{V_0(1 \pm \frac{\Delta V}{V_0})}{h_0(1 \pm \frac{\Delta L}{h_0})} = \frac{A_0(1 \pm \epsilon_v)}{1 \pm \epsilon_L}$~~

~~$A_c =$~~

$$A_c = \frac{V_0\left(1 \pm \frac{\Delta V}{V_0}\right)}{L\left(1 - \frac{\Delta L}{L}\right)}$$

$$\boxed{A_c = \frac{A_0(1 \pm \epsilon_v)}{1 - \epsilon_L}}$$

For Undrained test $\Delta V = 0$ $\quad \boxed{A_c = \frac{A_0}{1 - \epsilon_L}}$

⇒ Unconsolidated Undrained test (UU test)

- This test is quick takes only 5-7 minute.
- Expulsion of ~~water~~ pore water is not permitted in both the stages.
- used for short term.

⇒ Consolidated Drained test (CD test)

- Expulsion of pore water is ~~possible~~ permitted in both the cases.
- long term ~~analysis~~ stability analysis.
- Takes long time to ~~take~~ preform test.
- Best suited for SAND

⇒ Consolidated Undrained test (CU test)

- Expulsion of pw is permitted in first stage but not in 2nd stage.

⇒ Types of Triaxial test

Ist stage	2nd stage
1. Drainage allowed (Consolidated)	1. Volume change allowed (drained)
2. Drainage not allowed (Unconsolidated)	2. No volume change allowed (Undrained)

1. Consolidated drained test (CD test)

- Drainage is allowed in both stages.
- Pore water pressure doesn't build up.
- We get effective stress ~~to~~ parameter in this case.

Ist stage

Confining Pressure $= \sigma_3$, $PWP = 0$, $\sigma_3' = \sigma_1$

2nd stage

Confining Pressure $= \sigma_3$

deviator stress $= \sigma_d$

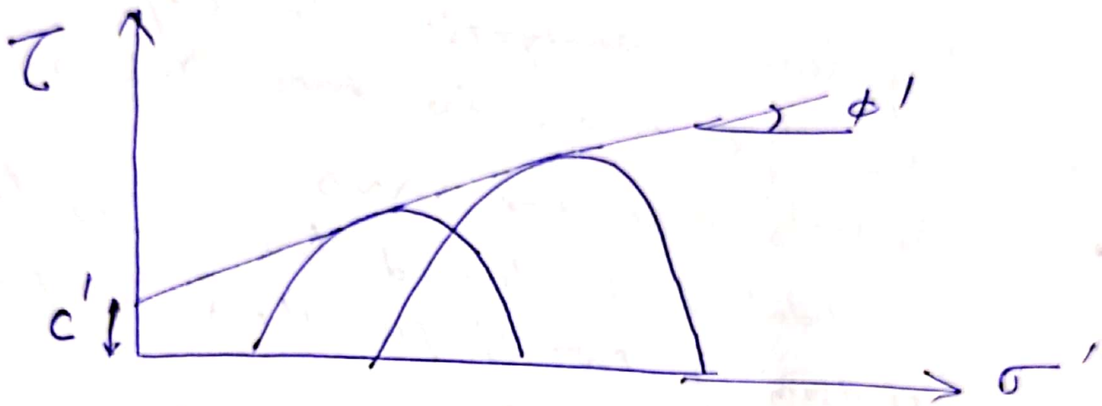
$\sigma_1 = \sigma_3 + \sigma_d$

PWP = 0

$\sigma_3' = \sigma_3$

$\sigma_1' = \sigma_1'$

Use: ① Analysis of gradual loading cond^n.
2. To check long term stability of embankment.



⇒ CU test

Ist stage → drainage Permitted

2nd stage → drainage not Permitted, volume change not allowed.

Ist stage

Confining stress $= \sigma_3$, $u = 0$, $\sigma_3' = \sigma_3$

2nd stage

Confining stress $= \sigma_3$

deviator stress $= \sigma_d$

Major $= \sigma_1 = \sigma_3 + \sigma_d$

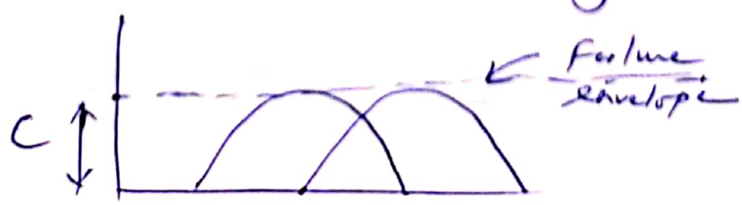
PWP = u

$\sigma_3' = \sigma_3 - u$

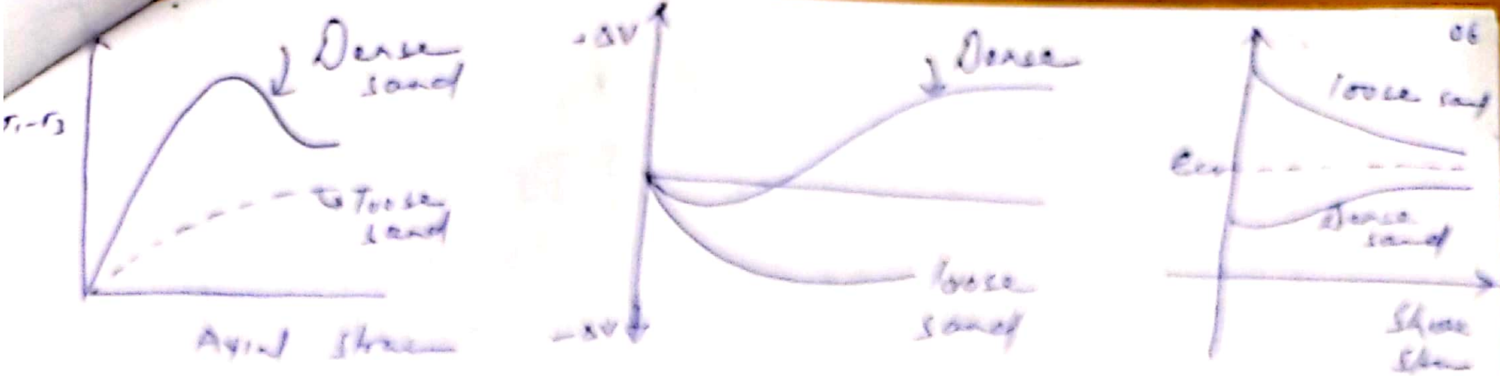
$\sigma_1' = \sigma_3' + \sigma_d$

⇒ UU test

- Drainage is not Permitted during Ist & 2nd stage.



→ Quick test
→ 15 min time taken

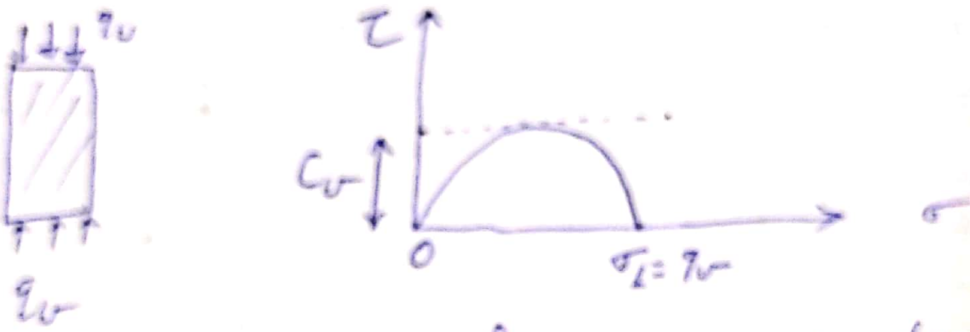


Results of CD test

⇒ Unconfined Compression test

→ Unconfined Compression test is a special case of triaxial in which cell pressure is zero ($\sigma_3 = 0$)

→ Used to test cohesive soil.



$$\sigma_1 = \cancel{\sigma_3 \tan^2\left(45+\frac{\phi}{2}\right)} + 2c\tan\left(45+\frac{\phi}{2}\right)$$

$$q_u = \sigma_1 = 2c\tan\left(45+\frac{\phi}{2}\right)$$

$$c = \frac{q_u}{2\tan\left(45+\frac{\phi}{2}\right)}$$

For Clay $\phi = 0$

$$c = \frac{q_u}{2}$$

→ $q_u = \frac{P}{A_f} = \frac{P}{\left(\frac{A_0}{1-\epsilon_1}\right)}$

P → Axial load applied

A_f → area of x-section at the time of failure

A_0 → Initial x-section area

Consistency	q_u (kPa)	
Very soft	0 - 24	0 - 25
Soft	24 - 48	25 - 50
Medium	48 - 96	50 - 100
Stiff	96 - 192	100 - 200
Very stiff	192 - 383	200 - 400
Hard	> 383	> 400

Vane shear test

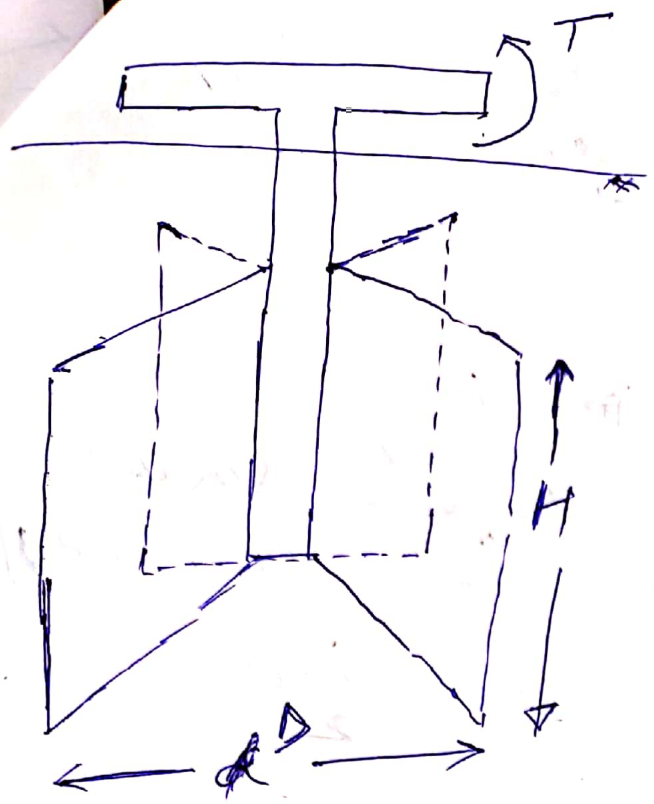
→ Vane shear test is suitable for soft saturated clay to find undrained shear strength.

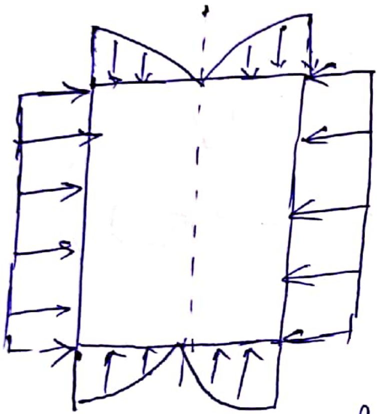
	LAB	Field
Height (H)	20 mm	10 - 20 cm
Dia (D)	12 mm	5 - 10 cm
Thickness of vane	0.5 - 1 mm	2 - 3 mm

The vane is pushed into the soil & Torque is applied by rotating the vane @ 6°/min. The vane is calibrated to a spring having torsional stiffness (K)

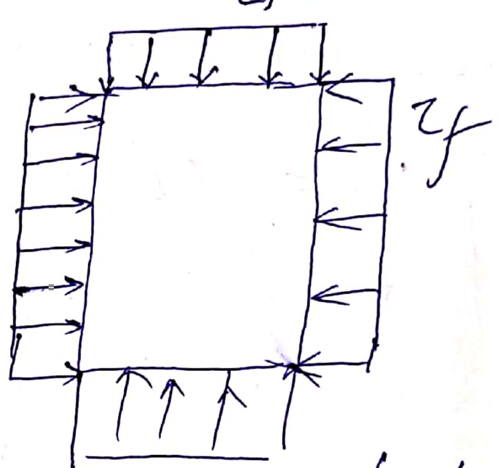
$$T = K \cdot \theta$$

θ → Angular rotation of vane at which complete shear failure occurs.





Before failure of soil sample



After failure of soil sample

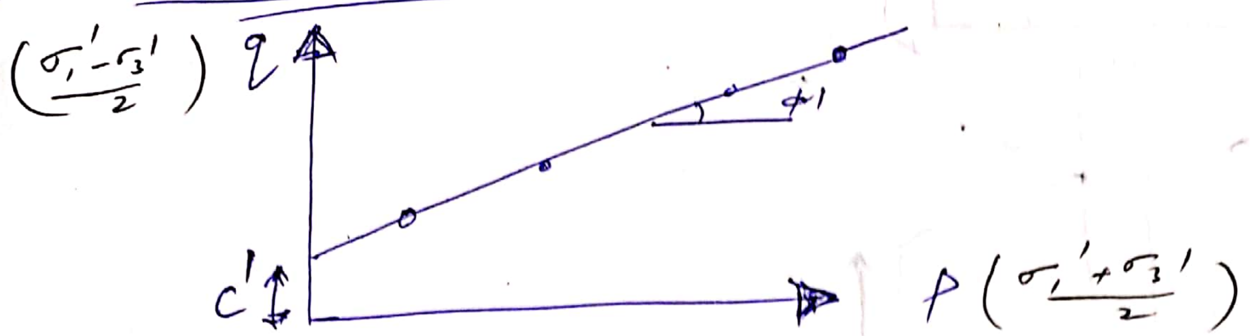
$$S = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{6}\right)}$$

↳ When shearing is done such that top ~~end~~ & bottom part partakes shear

$$S = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{12}\right)}$$

↳ Only bottom partakes shear

⇒ P-q ~~Pot~~ Plot



$$c' = c\cos\phi$$,

$$\tan\phi' = \sin\phi$$

$$\phi' = \tan^{-1}(\sin\phi)$$

Where $q = \frac{\sigma_1'-\sigma_3'}{2}$, $p = \frac{\sigma_1'+\sigma_3'}{2}$

⇒ Pore Pressure Coefficient (Skempton Pore Pressure Coefficient)

$$\Delta U = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$\Delta U = B\Delta\sigma_3 + AB(\Delta\sigma_1 - \Delta\sigma_3)]$$

$$B = \frac{\Delta U_1}{\Delta\sigma_3}$$

where, B → Skempton Pore Pressure Coefficient

$$\Delta U = B\Delta\sigma_3 + A_f(\Delta\sigma_1 - \Delta\sigma_3)$$

$$\Delta U = \Delta U_1 + \Delta U_2$$

$$\Delta U_1 = B\Delta\sigma_3$$

$$B = \frac{\Delta U_1}{\Delta\sigma_3}$$ → change in PWP due to an increase in cell pressure.

$$\Delta U_2 = A_f(\Delta\sigma_1 - \Delta\sigma_3)$$

$$A_f = \frac{\Delta U_2}{\Delta\sigma_1 - \Delta\sigma_3}$$ → Change in pore pressure due to increase in deviator stress ($\Delta\sigma_d$)

→ For a completely saturated soil, $B = 1$, $0 \le B \le 1$

→ For dry soil, $B = 0$, $B = f_n(S)$

→ Value of $A > 0$