Department of Civil Engineering
Katihar Engineering College, Katihar

Topic: Deflection
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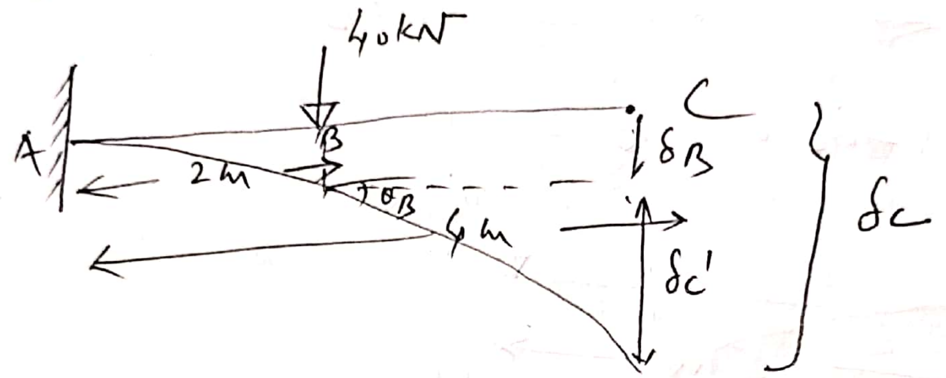
Lecture: 02.

Q-1 A cantilever beam of length 4m and a point is acting as shown in figure. Calculate deflection of the beam at the free end.



$$\delta_B = \frac{WL^3}{3EI}, \quad \theta_B = \frac{WL^2}{2EI}$$

$$\delta_B = \frac{40 \times 2^3}{3EI} = \frac{320}{3EI}$$

$$\theta_B = \frac{WL^2}{2EI} = \frac{40 \times 2^2}{2EI} = \frac{80}{EI}$$

$$\delta_C' = \theta_B \times L_{BC}$$

$$\delta_C' = \theta_B \times 2m$$

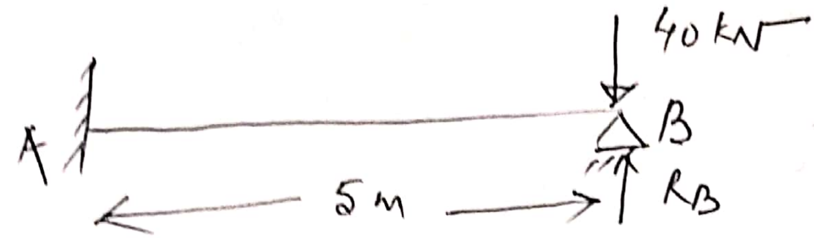
$$= \frac{80}{EI} \times 2 = \frac{160}{EI}$$

Deflection at the free end C $= \delta_B + \delta_C'$

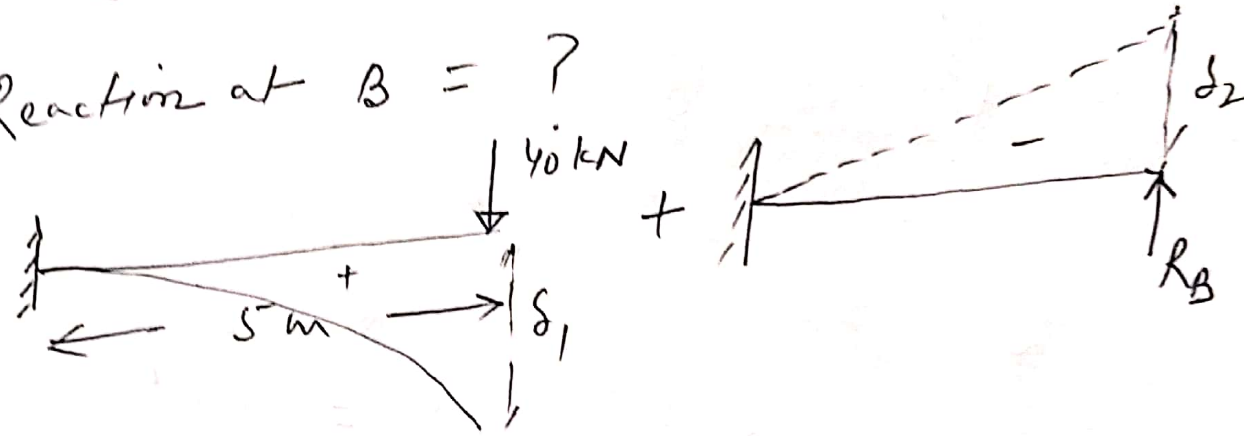
$$= \frac{320}{3EI} + \frac{160}{EI}$$

$$= \frac{320 + 480}{3EI} = \frac{800}{3EI}$$

Q-2.



Reaction at B = ?



$$\delta_1 = \frac{WL^3}{3EI}$$

$$= +\frac{40 \times 5^3}{3EI}$$

$$\delta_2 = \frac{R_B \times L^3}{3EI}$$

$$\delta_2 = -\frac{R_B \times 5^3}{3EI}$$

So net deflection at B should be zero

$$\frac{40 \times 5^3}{3EI} - \frac{R_B \times 5^3}{3EI} = 0$$

$$\boxed{R_B = 40\,kN}$$

Various methods :

1. Double integration Method
2. Macaulay's Method
3. Moment Area Method
4. Strain Energy Method
5. Castigliano's Theorem
6. Superposition Theorem

1. Double Integration Method

This method is useful when uniform bending moment eqn (when there is no sharp change) is applicable for entire beam

→ If Concentrated point load and Concentrated moment load is acting then Bending moment equation continuously change from move left to right or right to left. This method is not useful.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\int \frac{d^2y}{dx^2} = \frac{dy}{dx} = \theta = \int \frac{M}{EI} \cdot dx = \frac{M}{EI} \cdot x + c_1$$

Where θ is the slope.

If I once again differenciate

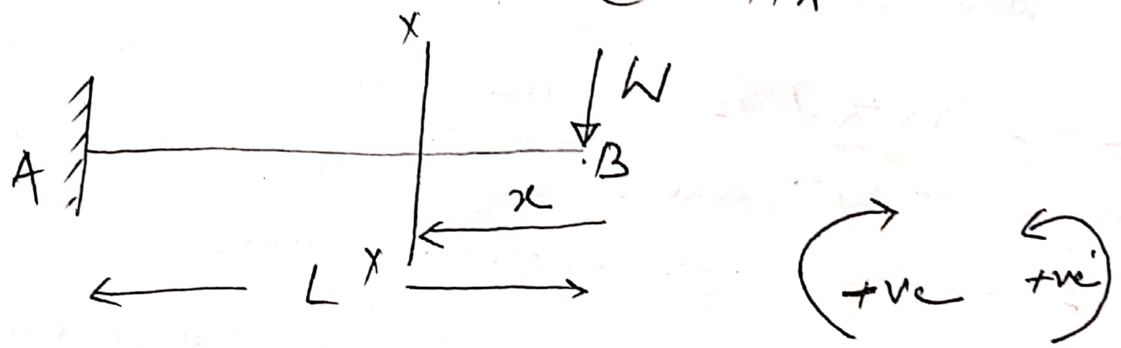
$$\int \frac{dy}{dx} = \int \left(\frac{M}{EI} x + C_1 \right) dx$$

$$y = \frac{M}{EI} x^2 + C_1 x + C_2$$

where $y \rightarrow$ deflection in the beam

C_1 & $C_2 \rightarrow$ are the constants and can be computed by applying boundary condition

P-1 :



We know that $$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$M_{x-x} = -W.x$$

$$\frac{d^2y}{dx^2} = \frac{-W.x}{EI}$$

$$\int \frac{d^2y}{dx^2} = \int \frac{-W.x}{EI} . dx$$

$$\frac{dy}{dx} = \theta = \frac{-Wx^2}{2EI} + C_1 \quad \text{---} (1)$$

If i integrate eqn 1

$$\int \frac{dy}{dx} = \int \left(\frac{-Wx^2}{2EI} + C_1 \right) dx$$

$$y = \frac{-Wx^3}{6EI} + C_1 x + C_2 \quad \text{--- } (2)$$

We know that

When $x = L$, $\theta = 0$

From 1

$$\frac{dy}{dx} = \theta = \frac{-Wx^2}{2EI} + C_1$$

$$0 = \frac{-WL^2}{2EI} + C_1$$

$$\boxed{C_1 = \frac{WL^2}{2EI}}$$

When $x = L$, $y = 0$

From eqn 2

$$y = \frac{-Wx^3}{6EI} + C_1 x + C_2$$

$$0 = \frac{-WL^3}{6EI} + \frac{WL^2}{2EI} \times L + C_2$$

$$C_2 = \frac{WL^3}{6EI} - \frac{WL^3}{2EI}$$

$$\boxed{C_2 = -\frac{WL^3}{3EI}}$$

From Equation 1

$$\theta = \frac{-Wx^2}{2EI} + C_1$$

$$\boxed{\theta = \frac{-Wx^2}{2EI} + \frac{WL^2}{2EI}}$$

Maxm Value of Slope or Slope at the free end (θ_B) $= \frac{-W \times 0^2}{2EI} + \frac{WL^2}{2EI}$

$$\boxed{\theta_B = \frac{WL^2}{2EI}}$$

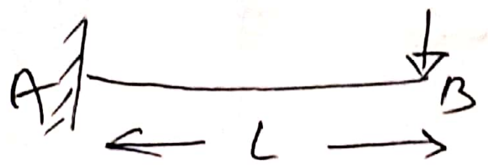
From Equation 2

$$y = \frac{-Wx^3}{6EI} + C_1 x + C_2$$

$$\boxed{y = \frac{-Wx^3}{6EI} + \frac{WL^2}{2EI} \cdot x - \frac{WL^3}{3EI}}$$

Max^m deflection at the free end $(\delta_B) = \frac{-WL^3}{3EI}$
$(x = 0)$

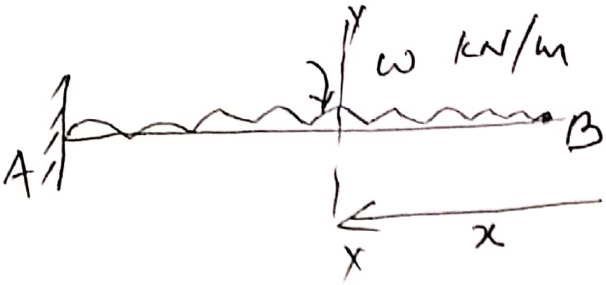
$$\boxed{\delta_B = \frac{WL^3}{3EI}}$$



$$\theta_B = \frac{WL^2}{2EI}$$

$$\delta_B = \frac{WL^3}{3EI}$$

P-2.



$$M_x = \frac{-wx^2}{2}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\theta = \frac{wL^3}{6EI}, \quad \delta = \frac{wL^4}{8EI}$$

<Happy Learning>