Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic : Slope and Deflection
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Lecture No : 03

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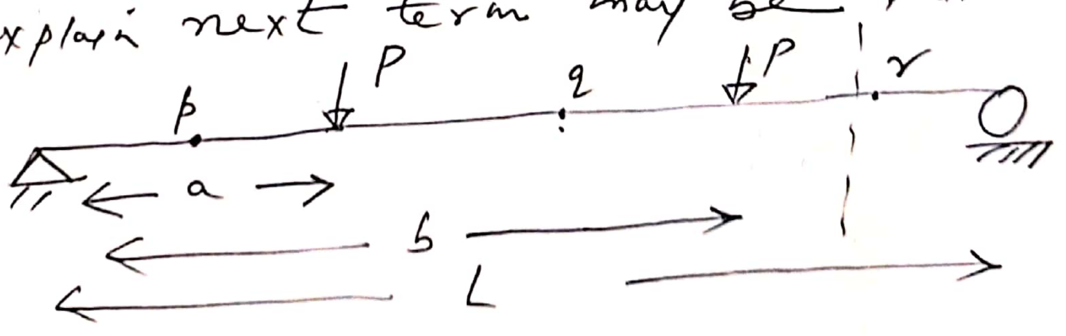
(2) Macaulay's Method

- Macaulay's method is improvement over double integration method.
- This method is useful when Equation of bending moment changes from one part to other part of span such as beam contains concentrated point load and Concentrated moment load.

⇒ Step to solve Macaulay's Method.

Step 1. Initially cut a section before the end of the beam either moving left to right or right to left in a beam then write down the equation of M_x

Use oblique sign that explain next term may be valid or not



$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

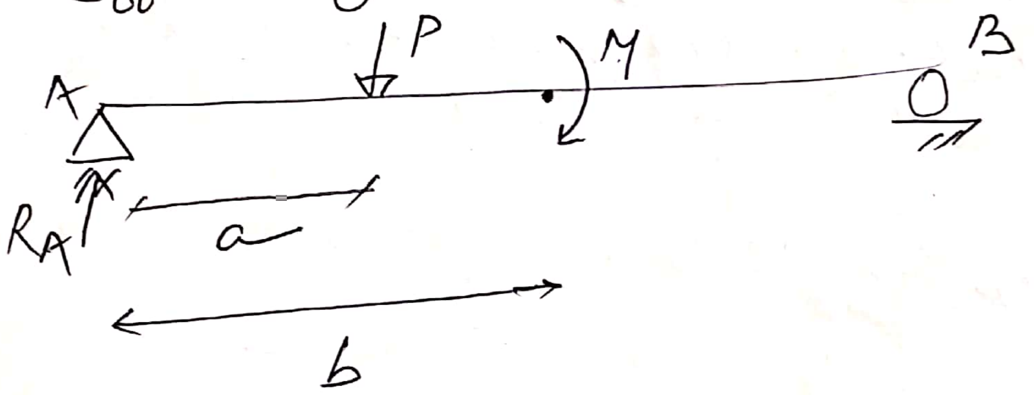
$$EI\frac{d^2y}{dx^2} = \underbrace{R_A \cdot x}_{\text{Ist term}} \,/\, \underbrace{-P(x-a)}_{\text{IInd term}} \,/\, \underbrace{-P(x-b)}_{\text{IIIrd term}}$$

$p \rightarrow$ $x < a$ (IInd & IIIrd term neglected)

$q \rightarrow$ $a < x < b$ (IIIrd term is neglected)

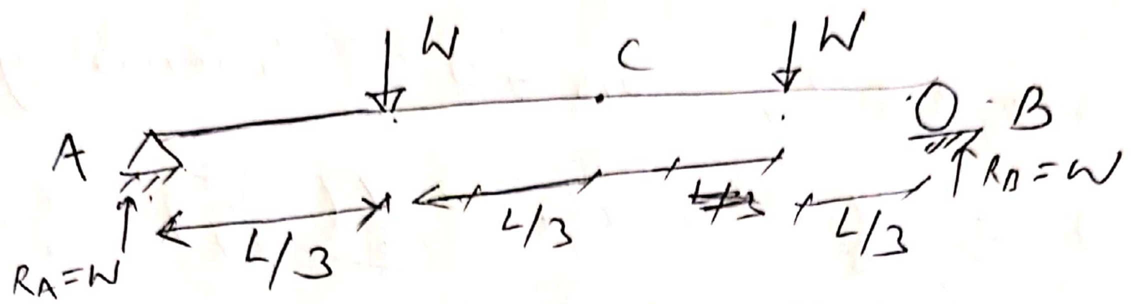
$r \rightarrow$ $x > b$ (No term neglected)

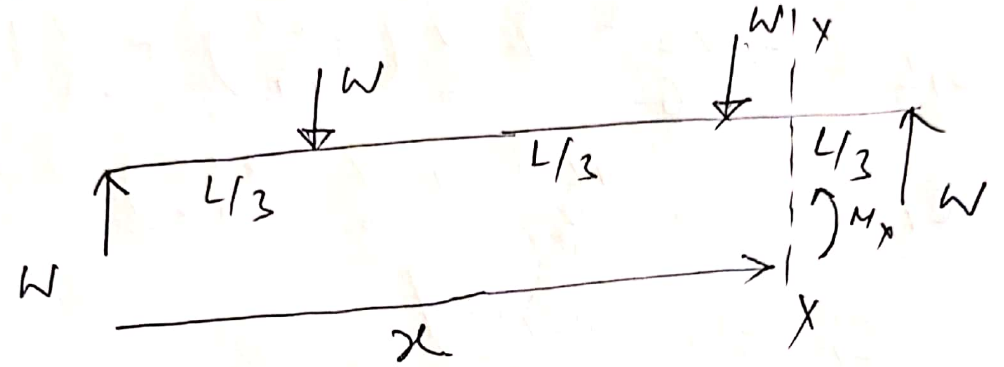
2. Effect of Concentrated Moment.



$$M_x = EI\frac{d^2y}{dx} = \underbrace{R_A \cdot x}_{\text{Ist term}} \,/\, + \underbrace{[-P(x-a)]}_{\text{IInd term}} \,/\, + \underbrace{[M(x-b)^0]}_{\text{IIIrd term}}$$

Problem-I. A simply supported beam with loading as shown in figure. Find out slope at point A & B and deflection at point C by using Macaulay's method.





$$M_x = W \cdot x - W(x - L/3) - W\left(x - \frac{2L}{3}\right)$$

$$EI\frac{d^2y}{dx^2} = \underbrace{W \cdot x}_{\text{Ist term}} \Big/ - \underbrace{W\left(x - \frac{L}{3}\right)}_{\text{IInd term}} \Big/ - \underbrace{W\left(x - \frac{2L}{3}\right)}_{\text{third term}}.$$

By Integration

$$EI\frac{dy}{dx} = \frac{Wx^2}{2} + C_1 \Big/ - \frac{W\left(x - \frac{L}{3}\right)^2}{2} \Big/ - \frac{W\left(x - \frac{2L}{3}\right)^2}{2} \quad \text{—— (1)}$$

Again Integrate Equation (1)

$$EI \cdot y = \frac{Wx^3}{6} + C_1 x + C_2 \Big/ - \frac{W}{2}\frac{\left(x - \frac{L}{3}\right)^3}{3} \Big/ - \frac{W}{2}\frac{\left(x - \frac{2L}{3}\right)^3}{3} \quad \text{—— (2)}$$

At $x = 0$, $\delta_A = 0$ (Neglect II^{nd} & III^{rd} term)

$$0 = C_2$$

$$\boxed{C_2 = 0}$$

At $x = L$, $\delta_B = 0$ (No term neglected)

$$0 = \frac{W}{6}L^3 + C_1 L - \frac{W}{6}\left(\frac{8L^3}{27}\right) - \frac{W}{6}\left(\frac{L^3}{27}\right)$$

$$C_1 L = -\frac{WL^3}{6} + \frac{W}{6}\left(\frac{8L^3}{27}\right) + \frac{W}{6}\left(\frac{L^3}{27}\right)$$

$$C_1 = -\frac{WL^2}{6} + \frac{W}{6}\left(\frac{8L^2}{27}\right) + \frac{W}{6}\left(\frac{L^2}{27}\right)$$

$$\boxed{C_1 = -\frac{WL^2}{9}}$$

From Eqn (1)

When $x = 0$ (Neglecting II^{nd} & III^{rd} term)

$$EI \cdot \theta_A = -\frac{WL^2}{9}$$

$$\boxed{\theta_A = -\frac{WL^2}{9EI}}$$

When $x = L$, θ_B [No term neglected)

$$EI \cdot \theta_B = \cancel{-\frac{WL^2}{9}}\ \frac{WL^2}{2} + \left(-\frac{WL^2}{9}\right) - \frac{W}{2}\left(\frac{4L^2}{9}\right) - \frac{W}{2}\left(\frac{L^2}{9}\right)$$

$$\theta_A = \frac{2WL^2}{18}$$

$$\boxed{\theta_B = \frac{WL^2}{9EI}}$$

For Computation of deflection at C

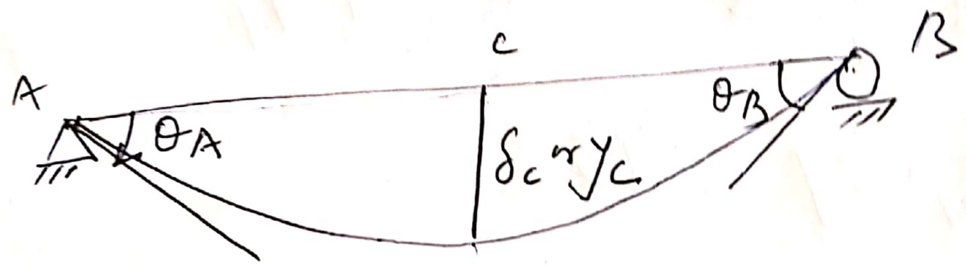
$$x = \frac{L}{3} + \frac{L}{6} = L/2$$

$$EI \cdot y_c = \frac{W}{6} \cdot \frac{L^3}{8} - \frac{WL^2}{9} \cdot \frac{L}{2} - \frac{W}{6}\left(\frac{L^3}{216}\right)$$

$$= \frac{WL^3}{48} - \frac{WL^3}{18} - \frac{WL^3}{1296}$$

$$= -\frac{23}{648} WL^3$$

$$\boxed{y_c = -\frac{23 \, WL^3}{648 \, EI}}$$

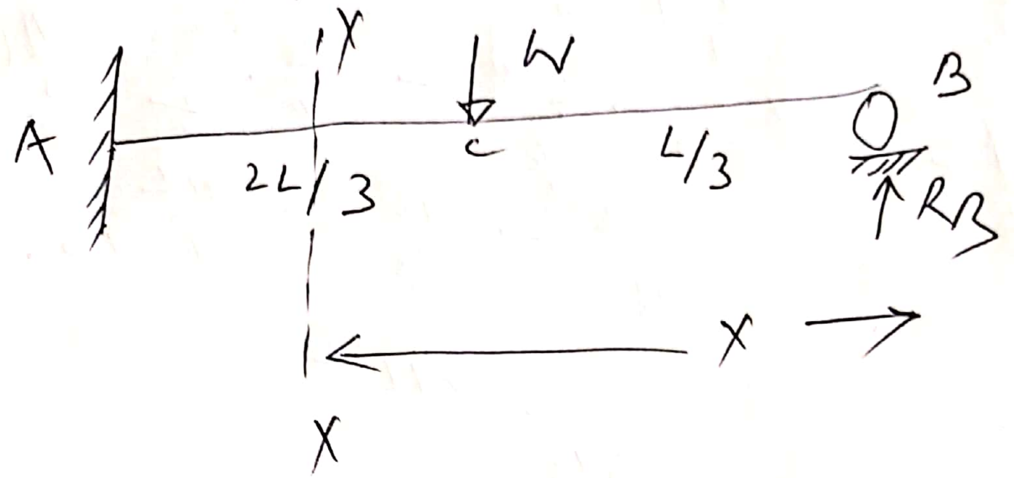


$$\theta_A = \frac{-WL^2}{9EI}$$

$$\theta_B = \frac{WL^2}{9EI}$$

$$y_c = \frac{-23}{648} \frac{WL^3}{EI}$$ (-ve sign indicates downward deflection)

Q-2. For the Propped Cantilever beam as shown in figure. Find out the Propped reaction & Slope at the point B by using Macaulay's Method.



$$M_x = EI\frac{d^2y}{dx^2} = -R_B \cdot x \Big/ + W\left(x - L/3\right)$$

$$EI\frac{dy}{dx} = \underbrace{-R_B\cdot\frac{x^2}{2} + C_1}_{1st} \Big/ + \underbrace{\frac{W\left(x-\frac{L}{3}\right)^2}{2}}_{2nd\ term} \quad \text{--- (1)}$$

Integrating Eqn (1)

$$EIy = -R_B\cdot\frac{x^3}{6} + C_1x + C_2 \Big/ + \frac{W}{6}\left(x-\frac{L}{3}\right)^3 \quad \text{--- (2)}$$

At $x = 0$, $\delta = 0$ (2nd term neglected)

$$\boxed{C_2 = 0}$$

At $x = L$, $\theta = 0$ (No term neglected)

$$0 = -\frac{R_B}{2}\cdot L^2 + C_1 + \frac{W}{2}\cdot\left(\frac{4L^2}{9}\right) \quad \text{--- (3)}$$

At $x = L$, $\delta_A = 0$

$$0 = -\frac{R_B}{2} \cdot \frac{L^3}{3} + C_1 L + \frac{W}{6}\left(\frac{8L^3}{27}\right) \quad \text{--- (4)}$$

$$C_1 - \frac{R_B L^2}{2} = -\frac{4WL^2}{18} \quad (\text{From (3)})$$

$$C_1 L - \frac{R_B L^3}{6} = -\frac{4WL^3}{81} \quad (\text{From (4)})$$

By solving this Equation you can compute the value of C_1 & Reaction R_B.

$\theta = ✓$

$y = ✓$

<Happy Learning>