Department of CIVIL Engineering ①

Katihar Engineering College, Katihar

Subject: Design of Concrete structure - I

TOPIC: Assignment - II Solution

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Q-1 (b)

$f_{ck} = 20 \text{ MPa}, \quad f_y = 415 \text{ MPa}$

$B = 300 \text{ mm}$

Effective depth (d) $= 600 - 25 - \frac{20}{2} = 565 \text{ mm}$

Area of tensile reinforcement (Ast) $= n \times \frac{\pi}{4} \times \phi^2$

$= 5 \times \frac{\pi}{4} \times 20^2$

$= 1570.80 \text{ mm}^2$

Percentage of reinforcement (p_t) $= \frac{100 A_{st}}{B d}$

$= \frac{1570.8 \times 100}{300 \times 565}$

$= 0.93\%$

p_t	τ_c (N/mm²)
0.75	0.56
0.93	—
1.00	0.62

$\tau_c |_{p_t = 0.93\%} = 0.56 + \frac{0.62 - 0.56}{(1.00 - 0.75)} (0.93 - 0.75)$

$\tau_c = 0.603 \text{ MPa}$

Critical shear capacity of Beam $= \tau_c \times Bd$

$= 0.603 \times 300 \times 565$

$= 102.109$ kN

Q-2 (a) $f_{ck} = 25$ MPa, $f_y = 415$ MPa

Parameters	Beam B_1	Beam B_2
(i) Width (B)	300 mm	300 mm
(ii) Effective depth (d)	$450 - 25 - 20/2$ $= 415$ mm	$450 - \frac{20}{2} - 25$ $= 415$ mm
(iii) τ_c	0.80 MPa	0.80 MPa
(iv) (τ_v) Nominal shear stress	$\frac{V_u}{Bd} = \frac{350 \times 10^3}{300 \times 415}$ $= 2.81$ MPa	$\frac{V_u}{Bd} = \frac{450 \times 10^3}{300 \times 415}$ $= 3.61$ MPa
(v) τ_{cmax}	3.1 MPa	3.1 MPa
(vi) IS Condition fulfilled	$\tau_c < \tau_v < \tau_{cmax}$ Design of shear reinforcement has to be provided	$\tau_v > \tau_{cmax}$ Beam will fail in shear
(vii) The strength of shear reinforcement (V_{us})	$(2.81 - 0.8) \times 300 \times 415$ $= 250$ kN	Beam has to be revised

Q-3 (d)

$f_{ck} = 20$ MPa, $f_y = 415$ MPa

Width (B) = 300 mm

Effective depth (d) $= 600 - 25 - \frac{20}{2}$

$= 565$ mm

$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.80$ mm²

$P_t = \frac{100 A_{st}}{Bd} = 0.93\%$

P_t	τ_c (N/mm²)
0.75	0.56
0.93	—
1.00	0.62

$$\tau_c \Big|_{p_t = 0.93\%} = 0.56 + \frac{(0.62 - 0.56)}{(1.00 - 0.75)} \times (0.93 - 0.75)$$

$\tau_c = 0.603$ MPa

Critical shear capacity (V_{us}) $= \tau_c \times Bd = 102.109$ kN

Nominal shear stress (τ_v) $= \frac{V_u}{Bd} = \frac{500 \times 10^3}{300 \times 565}$

$= 2.95$ MPa

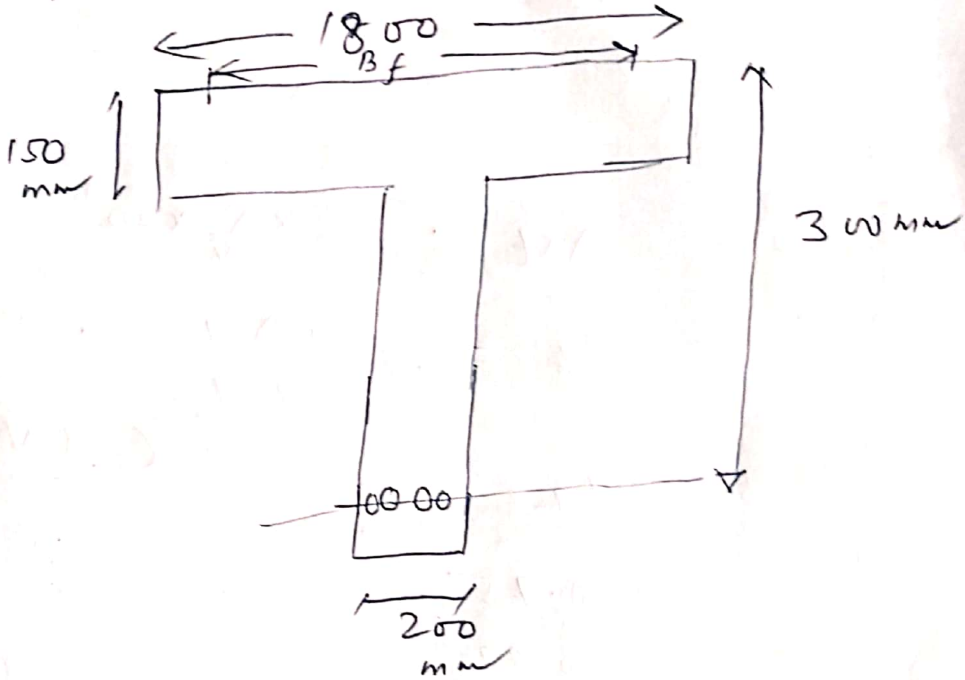
Since $\tau_v > \tau_{c\,max}$. Hence beam will fail in shear and no used of providing stirrups.

→ Increase depth would decrease the nominal shear stress.

Scanned with CamScanner

Q-4 (c) $f_{ck} = 25$ MPa, $f_y = 500$ MPa,

Distance b/w Points of zero moments in the Beam (L_o) = 3000 mm



For Beam + slab

Effective width of flange (B_f) $= \frac{L_o}{6} + b_w + 6D_f$

$= \frac{3000}{6} + 200 + 6 \times 150$

$= 1600$ mm

Q-5 (0.438%) M 20 Fe 415

$\sigma_{cbc} = 7 \ N/mm^2$ $\sigma_{st} = 230 \ N/mm^2$

$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \left(\frac{13.33 \times 7}{13.33 \times 7 + 230}\right) = 0.288$

$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 7} = 13.33$

Total Compression force (C) = Total tensile force

$\frac{1}{2} B x_c \sigma_{cbc} = \sigma_{st} \times A_{st}$

$A_{st} = \frac{1}{2} \frac{B x_c \sigma_{cbc}}{\sigma_{st}}$

% of steel (p_t) $= \frac{100 . A_{st}}{B . d}$

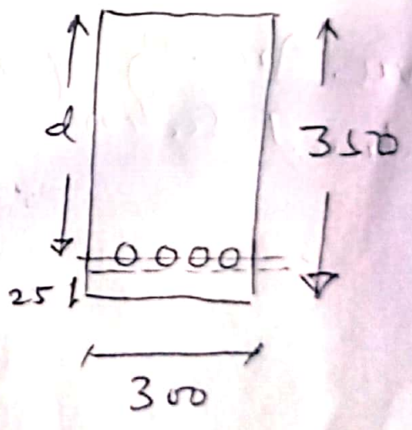
$= 100 \times \frac{1}{2} \frac{B \times k d \times \sigma_{cbc}}{B d \times \sigma_{st}}$

$= \frac{1}{2} \frac{k \, \sigma_{cbc}}{\sigma_{st}} \times 100$

$= \frac{1}{2} \times \frac{0.288 \times 7}{230} \times 100$

$= 0.438 \%$

Q-6. (C)



Effective depth (d) = D − Clear cover − $\frac{\phi}{2}$

$= 350 - 25 - \frac{20}{2}$

$= 315$ mm

Critical depth of NA (x_c) = k . d

$K = \frac{m \times \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} = \frac{9.33 \times 10}{9.33 \times 10 + 230} = 0.29$

$m = \frac{280}{3 \sigma_{cbc}} = \frac{280}{3 \times 10} = 9.33$

$X_c = 0.29 \times 315 = 91.35$ mm

For Actual depth of NA

$$\frac{B x_a^2}{2} = m A_{st} (d - X_a)$$

$$300 \times \frac{x_a^2}{2} = 9.33 \times 4 \times \frac{\pi}{4} \times 20^2 \times (315 - X_a)$$

$$150 x_a^2 + 11724.54 x_a - 3693231.518 = 0$$

$X_a = 122.62$ mm $> X_c$

Over reinforced Section

Note: This is an over reinforced section, and this section is not desirable.

Q-7. (254)

$B = 250$ mm } M20
$d = 317$ mm } Fe415

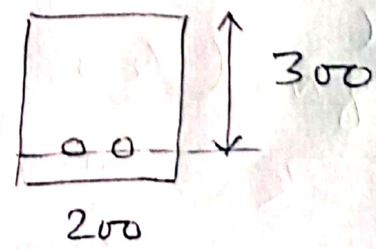
Lever Arm of Balanced Section $(Z) = (d - 0.42 X_{ulim})$

$= (317 - 0.42 \times 0.48 \times 317)$

$= 253.09$ mm

$\simeq 254$ mm

Q-8 (128)



$V_u = 80$ kN

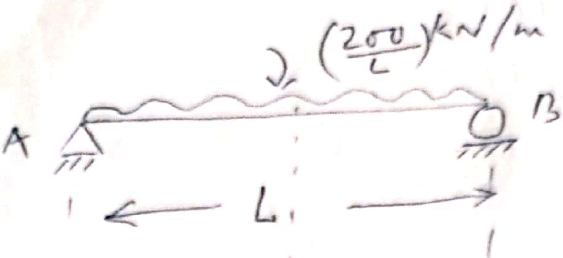
$T_u = 6$ kN-m.

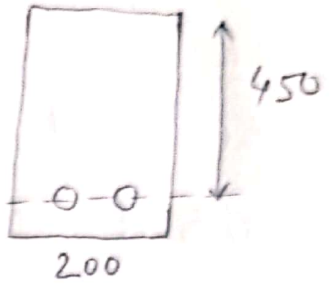
$V_e = V_u + 1.6 \frac{T_u}{B}$

$$V_e = 80 + 1.6 \times \frac{6}{0.2}$$

$$= 128 \text{ kN}$$

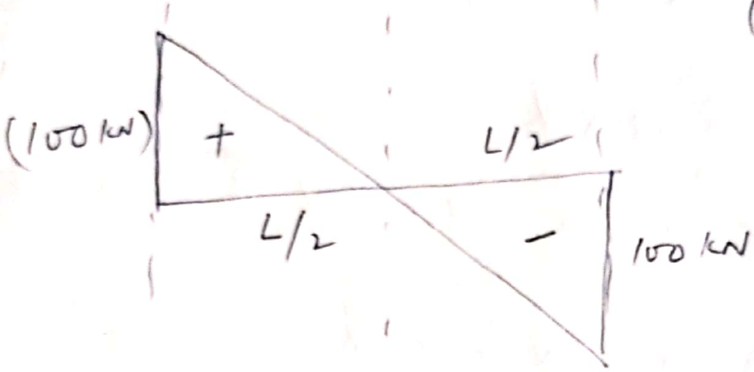
Q-9. (1.67 LSM) (1.11 WSM)





$R_A = R_B = 100$ kN

(ALL Dimensions are in mm)



Shear force Diagram (SFD)

Equation of shear force $(V_x) = 100 - \left(\frac{200}{L}\right) \times x$ [;]

when $x = 0$, $V_A = 100$ kN

When $x = L/2$, $V_A = 0$ kN

When $x = L$, $V_A = -100$ kN

Max^m Shear force $= 100$ kN

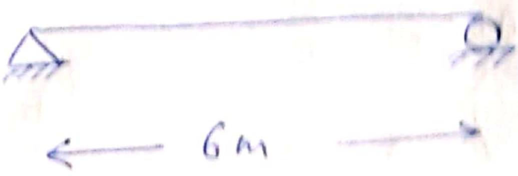
Factor Shear force $(V_u) = 100 \times 1.5 = 150$ kN

Nominal Shear stress $(\tau_v) = \frac{V_u}{Bd} = \frac{150 \times 10^3}{200 \times 450}$

$= 1.67$ N/mm^2 (By LSM)

$\tau_v = \frac{100 \times 10^3}{200 \times 450} = 1.11$ N/mm^2 (By WSM)

Q-10 (b)



DL = 15 kN/m

IL = 20 kN/m

Total Load = (15 + 20) = 35 kN/m

Factored BM = $\frac{1.5\,wL^2}{8} = \frac{1.5 \times 35 \times 6^2}{8}$

= 236.25 kN-m

$X_{ulim} = 0.48 \times d$

$M_u = 0.36\, f_{ck}\, B\, X_u\, (d - 0.42\, X_u)$

$236.25 \times 10^6 = 0.36 \times f_{ck} \times B \times X_{ulim}\,(d - 0.42 \times 0.48\, d)$

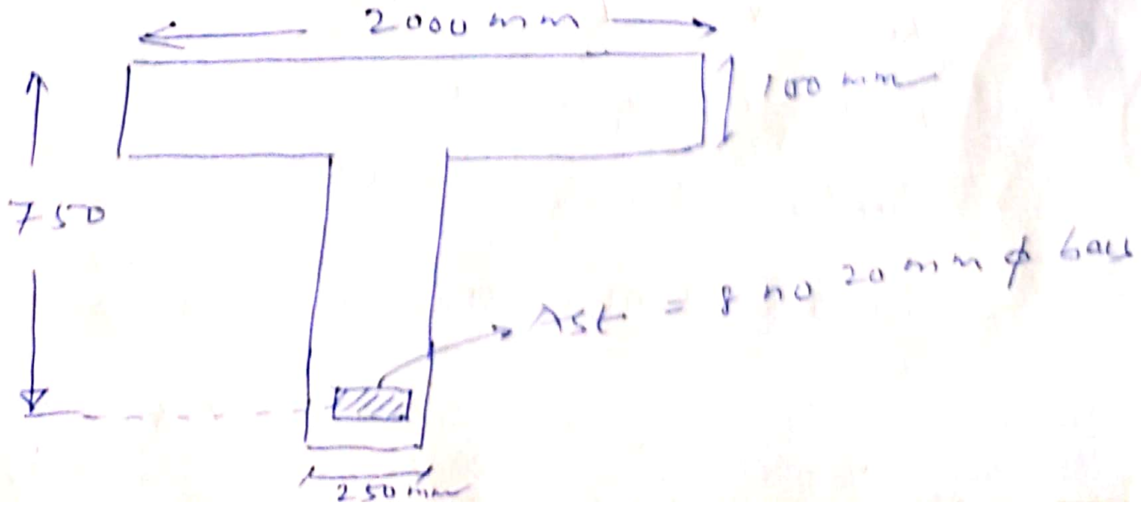
$236.25 \times 10^6 = 0.138\, f_{ck}\, B d^2$

$= 0.138 \times 25 \times B \times (2B)^2$

B = 257.73 mm

B ≃ 260 mm

Q-11 (b)



As the span of the beam is not mentioned, we can Consider it as Effective flange width

$$B = B_f = 2000 \text{ mm}$$

Assume Neutral Axis lies in flange ($x_u < D_f$)

$$0.36 f_{ck} B_f x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 \times \pi \times 20^2 \times 415}{4 \times 0.36 \times 25 \times 2000}$$

$$x_u = 50.412 \text{ mm} < 100 \text{ mm}$$ ok

$$x_{u\,lim} = 0.48 d = 0.48 \times 750$$

$$= 360 \text{ mm}$$

$x_u < x_{u\,lim}$ (Underreinforced section)

Moment of Resistance (MR) $= 0.36 f_{ck} B_f x_u (d - 0.42 x_u)$

$$= 0.36 \times 25 \times 2000 \times 50.412 \times (750 - 0.42 \times 50.412)$$

$$= 661349244.7 \text{ N-m}$$

$$= 661.35 \text{ kN-m}$$

Q-12. (778 mm²)

$B = 200$ mm

$d = 350$ mm

$BM = 24{,}000$ N-m $= 24$ kN-m

$c = 5 \text{ N/mm}^2$, $t = 140 \text{ N/mm}^2$

$m = 18$

Critical depth of NA $(x_c) = k.d$

$$k = \frac{m.\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{18.67 \times 5}{18.67\times5+140} = 0.40$$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3\times5} = 18.67$$

$$x_c = 0.40 \times 350 = 140 \text{ mm}$$

Actual depth of N.A.

$$\frac{B x_a^2}{2} = m.A_{st}(d - x_a)$$

$$200.\frac{x_a^2}{2} = 18.67 \times A_{st}(350 - x_a) \quad \text{—(}$$

$$MR_{Balance} = \underbrace{\tfrac{1}{2} C\, J\, K}_{Q}\; B d^2$$

$$= \frac{1}{2}\times 5\times\left(1-\frac{k}{3}\right)\times k \times B d^2$$

$$= 2.5 \times (0.40)\left(1 - \frac{0.40}{3}\right) \times 200 \times 350^2$$

$$= 20.83 \text{ kN-m}$$

Bending moment $(M) = 24$ kN-m

$M > MR_{balance}$ (over reinforced)

Ot

$$C_a = \sigma_{cbc} = 5 \text{ N/mm}^2$$

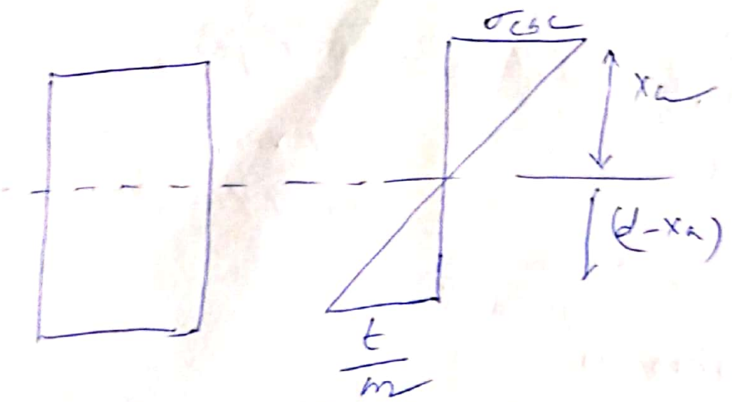
$$\frac{t_a}{m} < \frac{\sigma_{st}}{m} \Rightarrow t_a < \sigma_{st}$$

$$BM = B x_a \frac{\sigma_{cbc}}{2}\left(d - \frac{x_a}{3}\right)$$

$$24\times10^6 = 200.x_a \times \frac{5}{2}\left(350 - \frac{x_a}{3}\right)$$

$$\frac{x_a^2}{3} - 350x_a + 48000 = 0$$

$$x_a = 162.2 \text{ mm}$$



$$\frac{\sigma_{cbc}}{x_a} = \frac{(t_a/m)}{(d-x_a)}$$

$$t_a = \frac{\sigma_{cbc} \times m \times (d-x_a)}{x_a}$$

$$t_a = \frac{5 \times 18 \times (350 - 162.2)}{162.2}$$

$$t_a = 104.2 \text{ N/mm}^2$$

$$BM = t_a . A_{st} \left(d - \frac{x_a}{3}\right)$$

$$A_{st} = \frac{24 \times 10^6}{104.2\left(350 - \frac{162.2}{3}\right)} = 778 \text{ mm}^2$$

Q-13 (a)

For M25 grade of Concrete

$$\sigma_{cbc} = 8.5 \text{ N/mm}^2$$

long term modular ratio (m) $= \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 8.5}$

$$= 10.98 \simeq 11$$

Short term modular ratio (m) $= \frac{E_s}{E_c} = \frac{2 \times 10^5}{5000\sqrt{25}} = 8$

Q-14 (a)

For M25 Concrete

In LSM

direct Compression (strength) $= 0.4 f_{ck}$

$= 0.4 \times 25$

$= 10 \ N/mm^2$

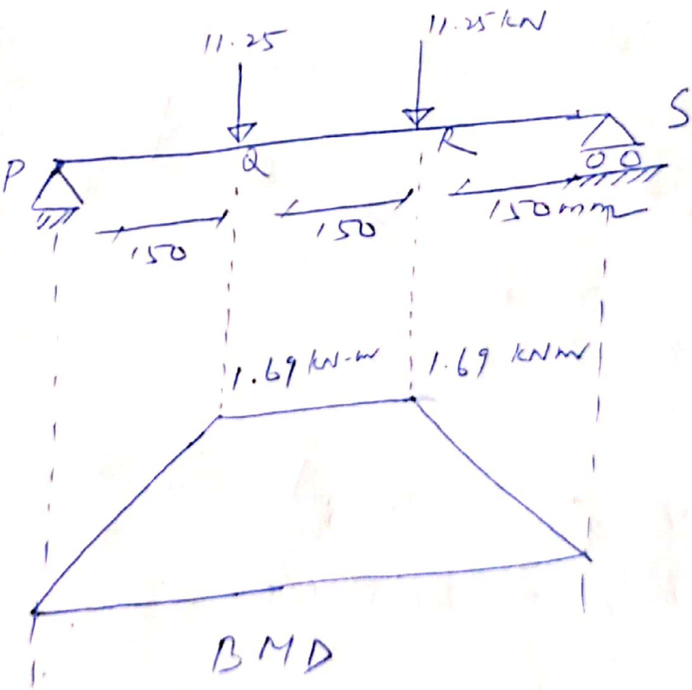
In bending compression strength $= 0.45 f_{ck}$

$= 0.45 \times 25 = 11.25 \ N/mm^2$

Strength in flexural tension $= 0.7 \sqrt{f_{ck}}$

$= 3.5 \ N/mm^2$

Q-15. (3)



150 × 150

Reaction at P and S = 11.25 kN

Moment at Q = $R_P \times \frac{150}{1000} = \frac{11.25 \times 150}{1000}$

$= 1.69$ kN-m.

Modulus of rupture (σ) $= \frac{M_{max} y}{I} = \frac{1.69 \times 10^6 \times 75 \times 12}{150 \times 150^3} = 3$ MPa

Q-18 (25.362 kNm)

(a) Effective depth $(d) = D - \text{Clear cover} - \frac{\phi}{2}$

$= 400 - 25 - \frac{20}{2}$

$= 365 \text{ mm}$

$A_{st} = 4 \times \frac{\pi}{4} \times 20^2$

$= 1256.6 \text{ mm}^2$

$A_{sc} = 2 \times \frac{\pi}{4} \times 12^2 = 226.2 \text{ mm}^2$

Area of tensile reinforcement, balanced section (A_{stb})

$$A_{stb} = \frac{0.36 \times 20 \times 0.48 \times 300 \times 365}{0.87 \times 415}$$

$= 1046.3 \text{ mm}^2$

Additional tensile reinforcement available $(A_{st2}) = A_{st} - A_{stb}$

$= 1256.6 - 1046.3$

$= 210.3 \text{ mm}^2$

Additional moment of resistance due to tensile reinforcement M_{ust2}

$= 0.87 f_y A_{st2} (d - d_c)$

$= 0.87 \times 415 \times 210.3 \times (365 - 31)$

$= 25.362 \text{ kN-m}$

Q-17 (353.1 N/mm²)

$f_{ck} = 20\ N/mm^2$, $f_y = 415\ N/mm^2$

$B = 300\ mm$, $D = 400\ mm$

$d = 400 - 25 - \frac{20}{2} = 365\ mm$

Strain in Concrete (ϵ_c) = 0.0035

Effective cover in Compression side (d_c) = $cc + \phi_c/2$

$= 25 + \frac{12}{2}$

$= 31\ mm$

Strain in Compression Steel (ϵ_{sc}) $= 0.0035\left(1 - \frac{d_c'}{x_{ulim}}\right)$

$= 0.0035\left(1 - \frac{31}{0.48 \times 365}\right)$

$= 0.0029$

Stress in Compression steel (f_{sc}) = ?

Strain (ϵ_{sc})	f_{sc}
0.00276	351
0.0029	—
0.00380	360

$$f_{sc}\Big|_{\epsilon_{sc} = 0.0029} = 351 + \frac{360 - 351}{(0.0038 - 0.00276)} \times (0.0029 - 0.00276)$$

$= 353.1\ N/mm^2$

Q-18 (110.138 kNm)

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$

$B = 300 mm$, $D = 400 mm$

$cc = 25 mm$

Effective depth (d) $= D - cc - \phi/2$

$= 400 - 25 - 20/2$

$= 365 mm$

$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.6 mm^2$

$A_{sc} = 2 \times \frac{\pi}{4} \times 12^2 = 226.2$

Moment of Resistance due to Concrete (M_{uc}) $= 0.138 f_{ck} B d^2$

$= 0.138 \times 20 \times 300 \times 365^2$

$= 110.138 kNm$

Q-19 (C)

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$

$B = 300 mm$,

$d = 600 - 25 - \frac{20}{2} = 565 mm$

$A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1570.8 mm^2$

$P_t = \frac{100 A_{st}}{Bd} = \frac{1570.8}{300 \times 565} \times 100 = 0.93\%$

$\tau_c |_{P_t = 0.93\%} = 0.56 + \frac{0.62 - 0.56}{(1 - 0.75)} (0.93 - 0.75)$

$= 0.603 N/mm^2$

Nominal shear Stress (τ_v) $= \frac{V_u}{Bd} = \frac{350 \times 10^3}{300 \times 565}$

$= 2.06 N/mm^2$

$\tau_c < \tau_v < \tau_{cmax} \longrightarrow$ Shear reinforcement has to be provided.

Strength of Shear reinforcement $(V_{us}) = \dfrac{(2.06 - 0.603) \times 300 \times 565}{10^3}$

$= 247$ kN

Spacing $(S_v) = \dfrac{0.87 f_y A_{sv} \cdot d}{V_{us}}$

$= \dfrac{0.87 \times 415 \times (2 \times \frac{\pi}{4} \times 12^2) \times 568}{247 \times 10^3}$

$= 186.8$ mm

Hence Provide 2-legged stirrups of diameter 12 mm @ 180 mm c/c

Q-20 (d) For vertical stirrups

$S_v = \dfrac{0.87 f_y A_{sv} \cdot d}{V_{us}}$

$\dfrac{V_{us}}{d} = \dfrac{0.87 f_y A_{sv}}{S_v}$

$\dfrac{V_{us}}{d} = 0.87 \times f_y \times \dfrac{A_{sv}}{S_v}$

<Happy Learning>