Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics

Topic: Assignment - II Solution

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Q-1 (c)

$$\text{Hoop stress} (\sigma_h) = \frac{pd}{2t}$$

$$100 = \frac{2 \times 800}{2 \times t}$$

$$t = 8 \text{ mm}$$

Q-2: (b)

$$\epsilon_L \text{ (longitudinal strain)} = \frac{\sigma_L}{E} - \nu \frac{\sigma_h}{E}$$

$$= \frac{p.d}{4tE} - \nu \frac{pd}{2tE}$$

$$= \frac{pd}{tE}\left[\frac{1}{4} - \frac{\nu}{2}\right]$$

$$\epsilon_L = \frac{pd}{4tE}(1-2\nu)$$

$$\text{Hoop strain } (\epsilon_h) = \frac{\sigma_h}{E} - \nu \frac{\sigma_L}{E}$$

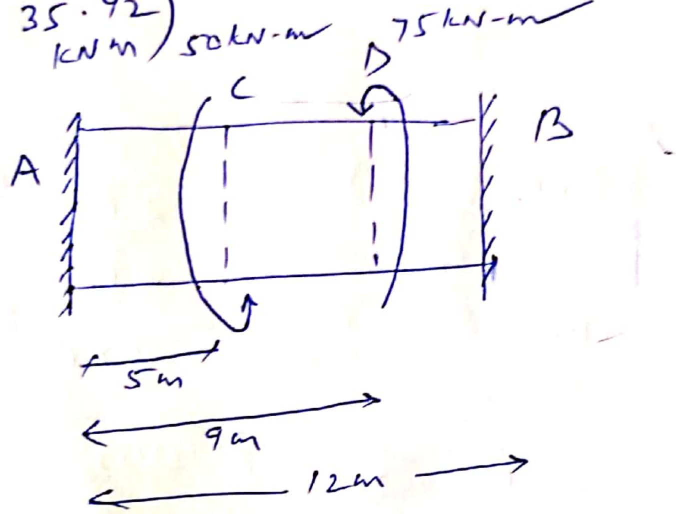
$$= \frac{p.d}{2tE} - \nu \frac{pd}{4tE}$$

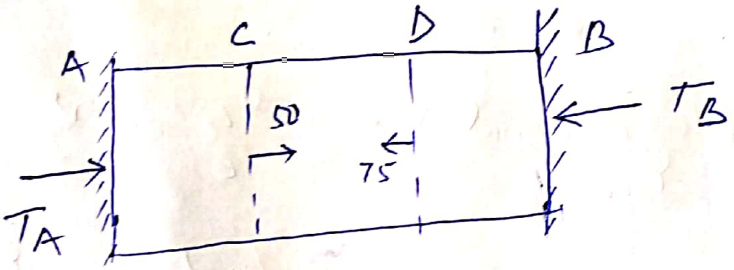
$$= \frac{pd}{t.E}\left[\frac{1}{2} - \frac{\nu}{4}\right]$$

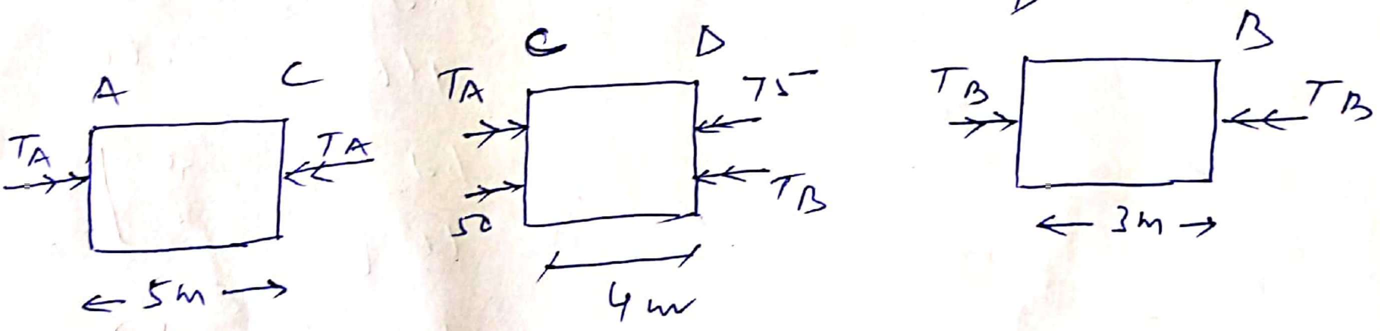
$$\frac{p.d}{4tE}\left[2-\nu\right]$$

$$\frac{\epsilon_L}{\epsilon_h} = \frac{\frac{pd}{4tE}(1-2\nu)}{\frac{pd}{4tE}(2-\nu)} = \frac{1-2\nu}{2-\nu}$$

Q-3 (10.42, 35.42 kNm)







Consider section CD

$$50 + T_A = T_B + 75$$

$$T_A - T_B = 25 \quad \text{—— (1)}$$

Since end A and B are fixed so,

$\theta_{AB} = 0$, θ_{AB} = Angle of twist b/w A and B.

$$\theta_{AB} = \theta_{AC} + \theta_{CD} + \theta_{DB}$$

$$\theta_{AB} = \theta_{AC} + \theta_{CD} + \theta_{DB}$$

$$\theta_B - \theta_A = \theta_{AC} + \theta_{CD} + \theta_{DB}$$

$$0 = \theta_{AC} + \theta_{CD} + \theta_{DB}$$

Using $\theta = \frac{TL}{NI_p}$

$$\frac{T_A \times 5}{NI_p} + \frac{4(50 + T_A)}{NI_p} + \frac{3T_B}{NI_p} = 0$$

$$5T_A + 200 + 4T_A + 3T_B = 0$$

$$9T_A + 3T_B = -200 \quad \text{—— (2)}$$

Solving Equation (1) & (2)

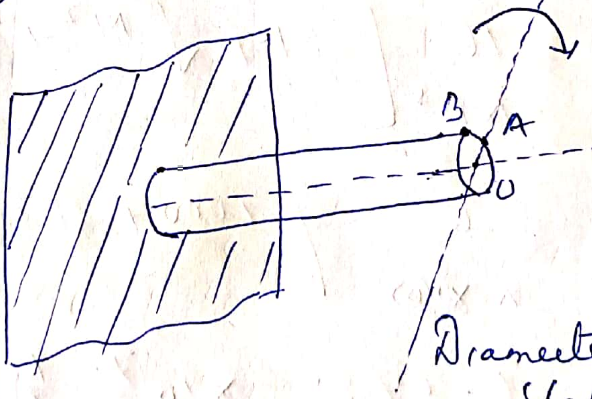
$$T_B = -35.42 \text{ kN-m}$$

$$T_A = -10.42 \text{ kN-m.}$$

$$\boxed{T_A = 10.42 \text{ kN-m}} \qquad \boxed{T_B = 35.42 \text{ kN-m}}$$

Q-4 (1648.95, -629.84, 1139.4)

Bending moment $(M) = 1000$ kg-m



Torque $(T) = 2000$ kg-m

Diameter of shaft $= 10$ cm $= 0.1$ m

A/c to Torsion Equation $\left[\frac{T}{I_p} = \frac{\tau}{r} = \frac{\tau_{max}}{R} = \frac{N\theta}{L} \right]$

$$\tau_{max} = \frac{16T}{\pi D^3} = \frac{16T}{\pi D^3}$$

A/c to Bending Equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{My}{I} = \frac{32M}{\pi D^3}$$

Let $\sigma_{P_1}/\sigma_{P_2}$ be the Major and Minor Principal stress.

$$\sigma_{P_1}/\sigma_{P_2} = \frac{\sigma + 0}{2} \pm \sqrt{\left(\frac{\sigma - 0}{2}\right)^2 + \tau^2}$$

$$= \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_{P_1}/\sigma_{P_2} = \frac{32M}{2 \cdot \pi D^3} \pm \sqrt{\left(\frac{32M}{2 \times \pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16M}{\pi D^3} \pm \sqrt{\left(\frac{16M}{\pi D^3}\right)^2 + \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

Major Principal stress $(\sigma_{P_1}) = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$

$$= \frac{16}{\pi \times (0.10)^3}\left[1000 + \sqrt{1000^2 + 2000^2}\right]$$

$$= 1.64895 \times 10^7 \text{ kg/m}^2$$

$$= 1648.95 \text{ kg/cm}^2$$

Minor Principal stress $(\sigma_{P_2}) = \frac{16}{\pi (0.1)^3}\left[1000 - \sqrt{1000^2 + 2000^2}\right]$

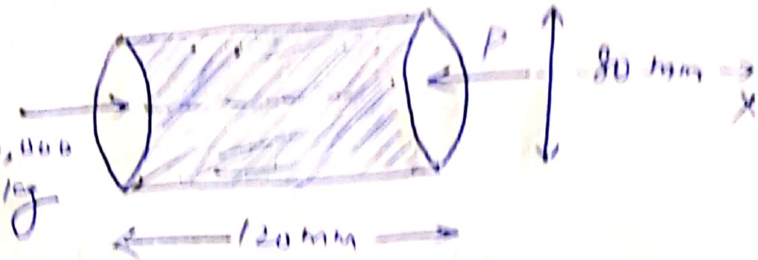
$$= -6.2984 \times 10^6 \text{ kg/cm}^2$$

$\sigma_{P_2} = -627.84 \ kg/cm^2$

$\tau_{max} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2}$ or $\frac{16}{\pi d^3}\sqrt{M^2+T^2}$

$\tau_{max} = 1139.4 \ kg/cm^2$

Q-5(a) ($0.1176 \ cm^3$)



Axial Compression load $(P) = 50,000$ kg

Bulk Modulus $(K) = 1.7 \times 10^6 \ kg/cm^2$

Poisson's ratio $(\nu) = 0.30$

Let ϵ_v be the volumetric strain

$$\epsilon_v = \frac{\Delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

In this case, $\sigma_y = 0, \ \sigma_z = 0$

$$\sigma_x = \frac{P}{A} = \frac{-50,000}{\frac{\pi}{4}\times 8^2} = -995.2 \ kg/cm^2$$

[-ve bcz its compression]

We also know that

$$E = 3K(1-2\nu)$$

$$E = 3 \times 1.7 \times 10^6 (1 - 2 \times 0.3)$$

$$E = 2.04 \times 10^6 \text{ kg/cm}^2$$

$$\epsilon_x = \frac{\sigma_x}{E} - \cancel{\frac{\nu \sigma_y}{E}} \to 0 - \cancel{\frac{\nu \sigma_z}{E}} \to 0$$

$$\epsilon_x = \frac{-995.2}{2.04 \times 10^6} = -4.878 \times 10^{-4}$$

$$\epsilon_y = -\nu \frac{\sigma_x}{E} = -0.3 \times (-4.878 \times 10^{-4})$$

$$= 1.4635 \times 10^{-4}$$

$$\epsilon_z = -\nu \frac{\sigma_x}{E} = 1.4635 \times 10^{-4}.$$

$$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= -4.878 \times 10^{-4} + 2 \times (1.4635) \times 10^{-4}$$

$$= -1.951 \times 10^{-4}$$

$$\frac{\Delta V}{V} = -1.951 \times 10^{-4}$$

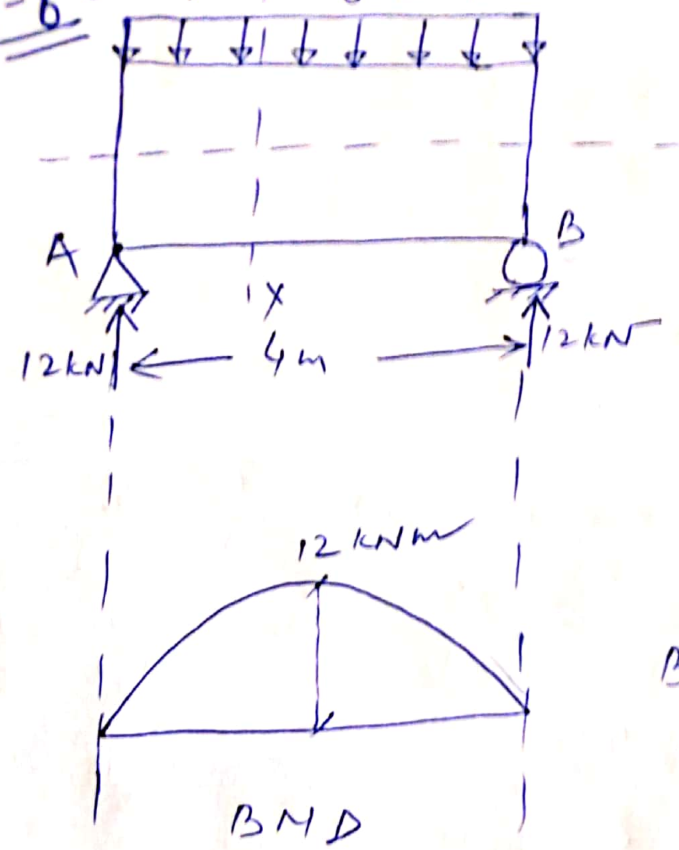
$$\Delta V = (-1.951 \times 10^{-4}) \times \frac{\pi}{4} \times (8^2) \times 12$$

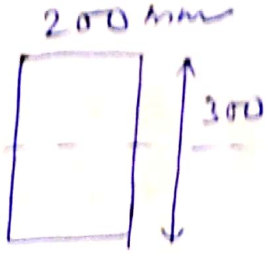
$$= -0.1176 \text{ cm}^3$$

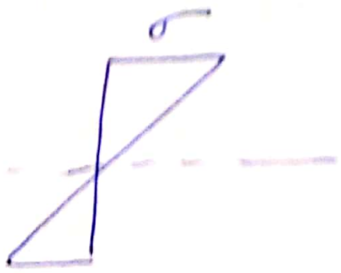
Change in volume is $-$ve.

Volume reduction $= 0.1176$ cm^3

Q-7 (4.04)







Bending stress diagram

Bending moment Equation $(M_x) = 12 \cdot x - 6 \cdot x \times \frac{x}{2}$

$= 12x - \frac{6x^2}{2}$

When $x = 0$, $M_A = 0$

When $x = 2m$, $M_{max} = 12 \times 2 - 3 \times 2^2$

$= 24 - 12$

$= 12$ kN-m

A/c to Bending Equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{M \cdot y}{I} = \frac{M}{Z} = \frac{12 \times 10^6 \times 6}{200 \times 300^2}$$

$= 4$ N/mm²

$\tau = 0.2$ N/mm²

$$\sigma_{P_1} = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

(Maxm principal stress) $= \frac{4}{2} + \sqrt{\left(\frac{4}{2}\right)^2 + 0.4^2} = 2 + \sqrt{4 + 0.4^2}$

$= 4.04$ N/mm²

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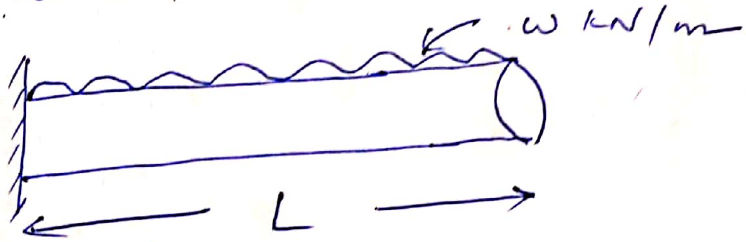
Q-7. (0.031, 32)

Let the intensity of loading & length of span be w kN/m and L respectively.



Maxm Curvature = 1.018592×10^{-6}/mm

Maxm Shear force = 1 kN

A/c to Bending Eqn

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{1}{R} = \frac{M}{EI}$$

Curvature

$$\text{Max}^m \text{ Curvature} = \frac{M_{max}}{EI}$$

$$1.018592 \times 10^{-6}/\text{mm} = \frac{M_{max}}{2 \times 10^5 \times \frac{\pi}{64}(2w)^4}$$

$$M_{max} = 16 \times 10^6 \text{ N-mm}$$

$$= 16 \text{ kNm}$$

M_{max} =

Maxm B.M in Cantilever shown above occurs at the fixed support of magnitude $wL^2/2$

$$\frac{wL^2}{2} = 16$$

$$wL^2 = 32 \quad \text{——— (1)}$$

Maxm shear force occurs at the fixed End of magnitude = wL

$$wL = 1 \quad \text{—— (2)}$$

Diving Eqn (1) by Eqn (2)

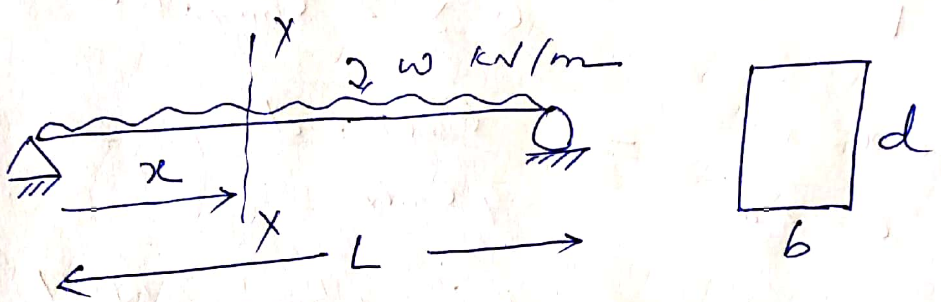
$$\frac{\frac{wL^2}{wL}}{} = \frac{32}{1}$$

$$\boxed{L = 32\ m}$$

$$w = \frac{1}{32}\ kN/m = 0.031\ \frac{kN}{m}$$

$$w = \frac{1000}{32} = 31.25\ N/m$$

Q-8 (b)



Let w be the load intensity

Given $\tau_{max} = 1.5 \frac{V}{Bd} = \frac{3V}{2Bd}$ —— (1)

Maxm bending stress (σ_{max}) = $\frac{M \times d/2}{\frac{bd^3}{12}} = \frac{6M}{bd^2}$ —— (2)

Let M_x be the Bending Moment at X-X & V be the shear force at X-X

$$M = \frac{wL}{2}.x - \frac{wx^2}{2}$$

$$V = \frac{wL}{2} - wx$$

$$\frac{M}{V} = \frac{\left(\frac{WL}{2}x - \frac{wx^2}{2}\right)}{\frac{WL}{2} - wx} = \frac{Lx - x^2}{L - 2x}$$

$$\frac{M}{V} = \left(\frac{Lx - x^2}{L - 2x}\right) \quad \text{--- (3)}$$

From (1) $\quad V = \dfrac{2\,\tau_{max} \cdot bd}{3}$

From (2) $\quad M = \dfrac{\sigma_{max} \cdot bd^2}{6}$

$$\frac{M}{V} = \frac{\sigma_{max} \cdot bd^2 \times 3}{6 \times 2\,\tau_{max} \cdot bd}$$

$$\frac{Lx - x^2}{L - 2x} = \frac{\sigma_{max} \cdot d}{4\,\tau_{max}} \quad \text{--- (4)}$$

Max^m value of Bending stress will occur at mid span.

$$\frac{wL^2}{8} = \sigma_{max} \cdot \frac{bd^2}{6}$$

$$\tau_{max} = \frac{3}{2} \frac{V}{bd}$$

$$\tau_{max} = \frac{3}{2} \times \frac{WL}{2 \cdot bd} = \frac{3}{4} \frac{WL}{bd}$$

$$\tau_{max} \cdot \frac{2}{3} (bd) = \frac{WL}{2}$$

$$\frac{Lx - x^2}{L - 2x} = \frac{wL^2/8}{\frac{WL}{2}} = \frac{L}{4}$$

$$L^2 - 2Lx = 4Lx - 4x^2$$

$$4x^2 - 6Lx + L^2 = 0$$

$$x = \frac{6L \pm \sqrt{36L^2 - 4\times4\times L^2}}{8}$$

$$x = \frac{6L \pm \sqrt{20L^2}}{8}$$

$$x = (6 \pm 2\sqrt{5})\frac{L}{8}$$

$$x = 1.309L, \quad 0.191L$$

the Critical length will be $0.191L$ from end A.

Q-9 (4, 3)

$$y = \frac{1}{EI}\left(-2x^3 + \frac{x^4}{6} + 36x\right)$$

$$\frac{dy}{dx} = \frac{1}{EI}\left(-6x^2 + \frac{4x^3}{6} + 36\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left(-12x + \frac{12x^2}{6}\right)$$

$$EI\frac{d^2y}{dx^2} = -12x + 2x^2$$

$$EI\frac{d^2y}{dx^2} = M$$

$$M = 2x^2 - 12x$$

$$\frac{dM}{dx} = 4x - 12 = V_x$$

$$\frac{dV_x}{dx} = 4 = \omega$$

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Finding the Position Where $V_x = 0$

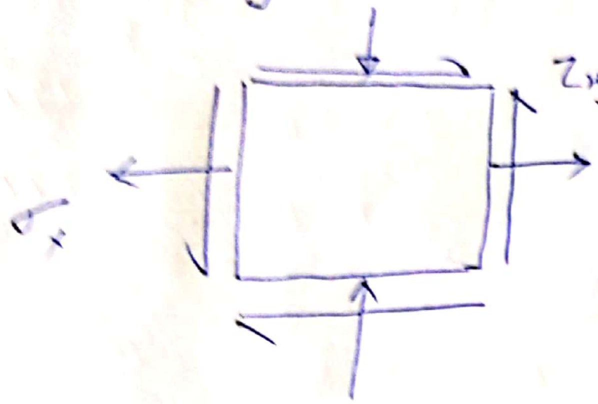
$4x - 12 = 0$ $\boxed{x = 3\,m}$

Q-10 (b)

$\sigma_x = 10,000\ N/mm^2$

$\sigma_y = -6000\ N/mm^2$

$\tau_{xy} = 8000\ N/mm^2$



Let $\sigma_{p_1} / \sigma_{p_2}$ be the Major & Minor Principal Stress.

$$\sigma_{p_1}/\sigma_{p_2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{10,000 - 6,000}{2} \pm \sqrt{\left(\frac{-6000 - 10000}{2}\right)^2 + (8,000)^2}$$

$$= 2000 \pm \sqrt{(8000)^2 + (8000)^2}$$

$$= 2000 \pm 11313.71$$

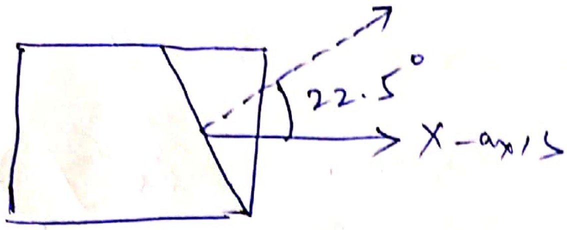
$\sigma_{p_1} = 13313.71\ N/mm^2$

$\sigma_{p_2} = -9313.71\ N/mm^2$

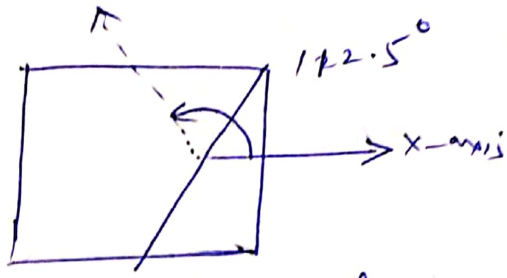
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 8000}{16000} = 1$$

$$2\theta_p = 45^\circ$$

$$\boxed{\theta_p = 22.5^\circ}, \text{ or } \boxed{112.5^\circ}$$



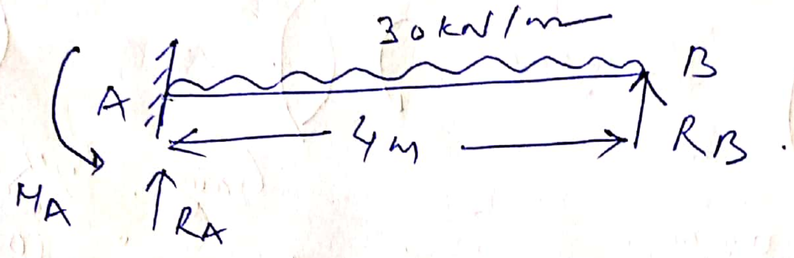
Major Principal Plane



Minor Principal Plane

Q-11 (0.115, 60)

$$EI = 2\times10^4 \text{ kN-m}^2$$

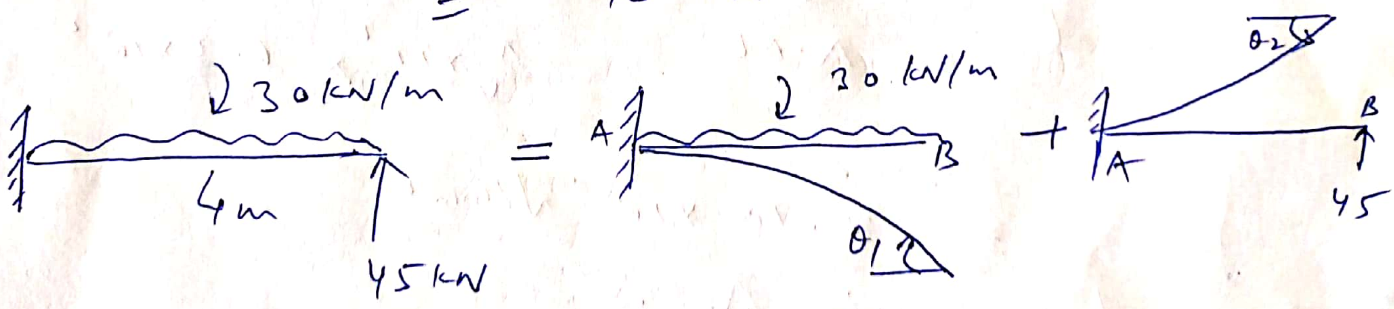


$$\delta_B = 0$$

$$\frac{wL^4}{8EI} - \frac{R_B L^3}{3EI} = 0$$

$$R_B = \frac{3wL}{8} = \frac{3\times30\times4}{8}$$

$$= 45 \text{ kN}$$



$$\theta_1 = \frac{wL^3}{6EI} = \frac{30\times4^3}{6\times2\times10^4} = \frac{2}{125} \text{ radian (Clockwise)}$$

$$\theta_2 = \frac{R_B L^2}{2EI} = \frac{45\times4^2}{2\times2\times10^4} = \frac{9}{500} \text{ radian (Anti clock wise)}$$

(14)

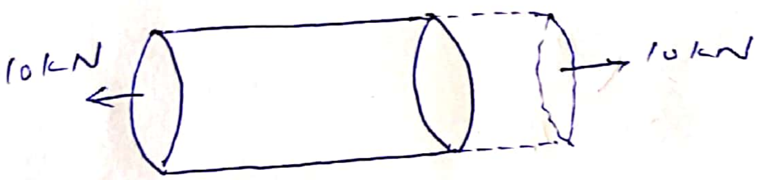
$\theta_B = \theta_2 - \theta_1 = \frac{1}{500} = 0.002$ radians (anticlockwise)

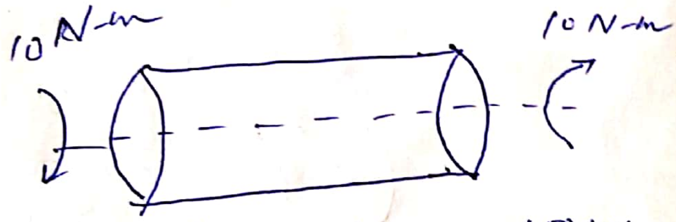
Moment at A $= 30 \times 4 \times \frac{4}{2} - 45 \times 4$

$= 60$ kNm (Hogging)

$\theta_B = 0.002 \times \frac{180}{\pi} = 0.115°$

Q-12 (2.10×10^5, 1.373×10^5, 8.44×10^4, 0.245)





Gauge length = 150 mm

$\theta = 0.5° = \left(\frac{0.5\pi}{180}\right)$ radian

$= 8.73 \times 10^{-3}$

dia = 12 mm

$I_p = \frac{\pi}{32} \times 12^4 = 2.0357 \times 10^3$ mm⁴

Gauge length = 150 mm

$\Delta L = 6.31 \times 10^{-2}$ mm

dia = 12 mm

$$\Delta = \frac{PL}{AE}$$

$$E = \frac{PL}{A \cdot \Delta L} = \frac{(10 \times 10^3) \times 150}{\frac{\pi}{4} \times 12^2 \times 6.3 \times 10^{-2}}$$

$$= 2.10 \times 10^5 \text{ N/mm}^2$$

Again from Torsion Data

$$\theta = \frac{TL}{NI_p} = \frac{(10 \times 10^3) \times 150}{(8.73 \times 10^{-3}) \times (2.03575 \times 10^3)}$$

$$N = 8.44 \times 10^4 \text{ mm}^2$$

$$E = 2N(1+\nu)$$

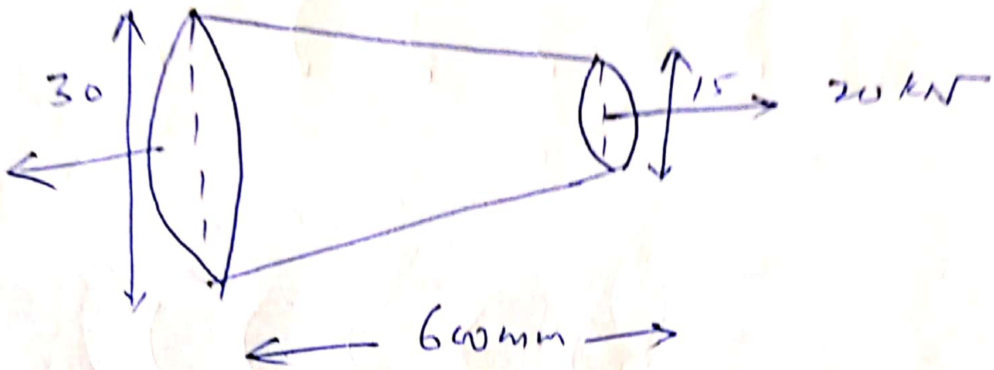
$$\nu = \left(\frac{E}{2N} - 1\right) = 0.245$$

Again

$$E = 3K(1-2\nu)$$

$$K = \frac{E}{3(1-2\nu)} = 1.373 \times 10^5 \ \frac{N}{mm^2}$$

Q-13 (0.1697)



$$\Delta = \frac{4PL}{\pi E D_1 D_2} = \frac{4 \times 20 \times 10^3 \times 600}{\pi \times 15 \times 30 \times 200 \times 10^3}$$

$$\boxed{\Delta = 0.1697 \text{ mm}}$$

Q-14 (Marks to all)

$\epsilon_x = +800$, Take $\theta = 60°$

$\epsilon_y = -1000$, $\phi_{xy} = -600$

$$\epsilon_n = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \phi_{xy} \sin\theta \cos\theta$$

$$= 800 \cos^2 60 - 1000 \sin^2 60 + (-600) \sin 60 . \cos 60$$

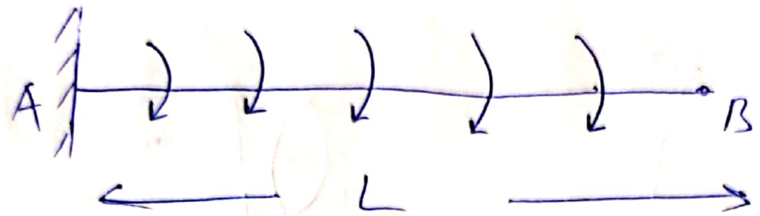
$$= -809.81 \text{ (Contraction}$$

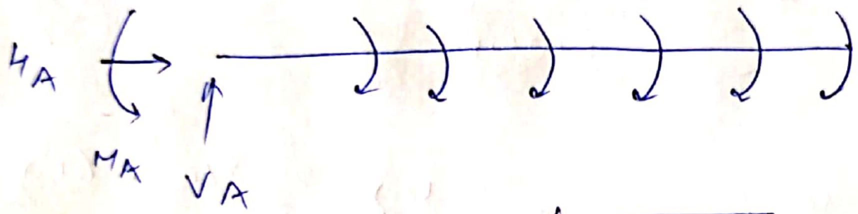
$$\frac{\phi_{x'y'}}{2} = (\epsilon_y - \epsilon_x)\sin\theta\cos\theta + \frac{\phi_{xy}}{2}(\cos^2\theta - \sin^2\theta)$$

$$= -629.42$$

$$\phi_{x'y'} = -1258.84$$

Q-15 (0, ML)





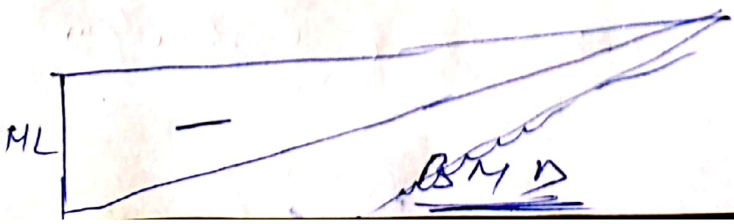
$$\boxed{H_A = 0}$$

$$\boxed{V_A = 0}$$

$$-M_A + M \times L = 0$$

$$\boxed{M_A = ML}$$

SFD



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