Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Slope and Deflection
Lecture: 04
Instructor: Prof. RASHID MUSTAFA

(3) Strain Energy Method :

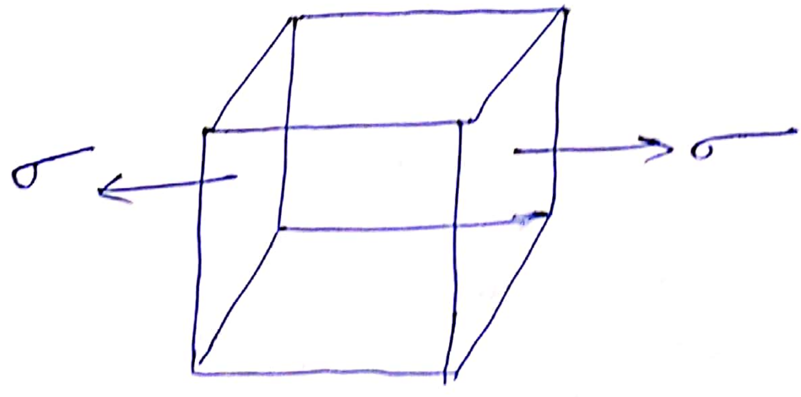
Assumptions

(i) No Loss of energy, hence External work done by load is equal to the internal energy stored which is called strain energy.

(ii) The load is such that the stresses induced never exceed the elastic limit.

(iii) Self weight of the member is neglected.

Strain Energy stored in a member.



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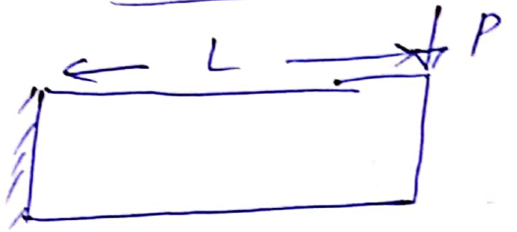
Resilence (strain energy density) $= \dfrac{\sigma^2}{2E} \times \text{Volume}$

$$= \int_0^L \frac{\sigma^2}{2E} \cdot dV$$

$\Rightarrow$ Strain Energy due to bending



A/c to Bending Equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

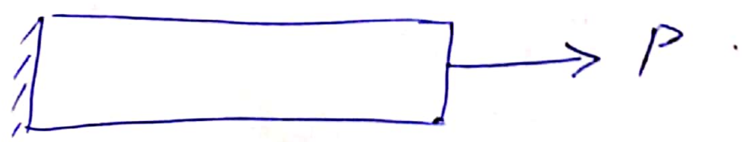
$$\sigma = \frac{M \cdot y}{I}$$

$$SE = \int_0^L \frac{M_x^2 \cdot y^2}{I^2 \cdot 2E} \cdot dA \cdot dx$$

$$= \int_0^L \frac{M_x^2 \times I}{2EI^2} \cdot dx$$

$$\boxed{SE = \int_0^L \frac{M_x^2}{2EI} \cdot dx}$$

$\Rightarrow$ Strain Energy due to Axial load.



$$SE = \frac{\sigma^2}{2E} \times \text{Volume} = \frac{P^2}{A^2 \cdot 2E} \times A \cdot L$$

$$= \frac{P^2 L}{2AE}$$

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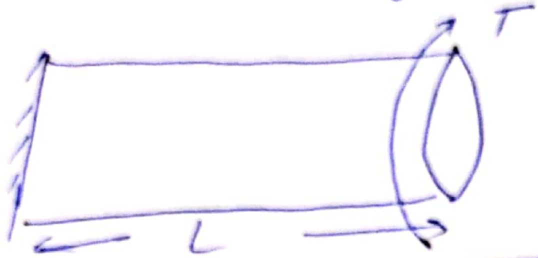
$$\boxed{SE = \int_0^L \frac{P_x^2 \, dx}{2AE}}$$

Where $AE \rightarrow$ Axial Rigidity

$\frac{AE}{L} \rightarrow$ Axial stiffness.

$\Rightarrow$ Strain Energy due to torsion.



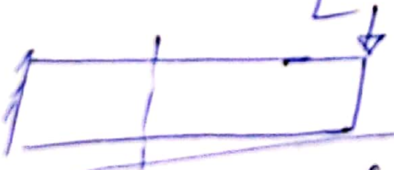
$$\boxed{S.E = \int_0^L \frac{T_x^2 \, dx}{2NI_p}}$$

Where $T_x \rightarrow$ Torsion at a distance x

$NI_p \rightarrow$ Torsional Rigidity

$\frac{NI_p}{L} \rightarrow$ Torsional stiffness.

$\Rightarrow$



$$\boxed{SE = \int_0^L \frac{M_x^2 \, dx}{2EI}}$$

$$\boxed{\text{SE due to Shear} = \int_0^L \frac{V_x^2 \, dx}{2AN}}$$

Where $V_x \rightarrow$ Shear force at a distance x

$AN \rightarrow$ Shear rigidity

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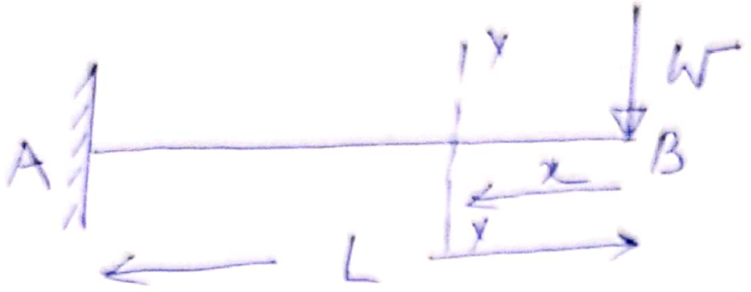
P-1. A cantilever Beam of length L & flexural rigidity EI. Beam contains a point load W at its free end. Compute the Strain Energy due to Bending



Strain Energy due to bending $= \int_0^L \frac{M_x^2 dx}{2EI}$

Equation of Bending moment at X-X $(M_x) = -W \cdot x$

$$S.E = \int_0^L \frac{(-W \cdot x)^2 \cdot dx}{2EI}$$

$$= \int_0^L \frac{W^2 x^2 dx}{2EI}$$

$$= \frac{W^2}{2EI} \int_0^L x^2 dx$$

$$= \frac{W^2}{2EI} \left[\frac{x^3}{3} \right]_0^L$$

$$= \frac{W^2}{2EI} \left[\frac{L^3}{3} \right]$$

$$\boxed{SE = \frac{W^2 L^3}{6EI}}$$

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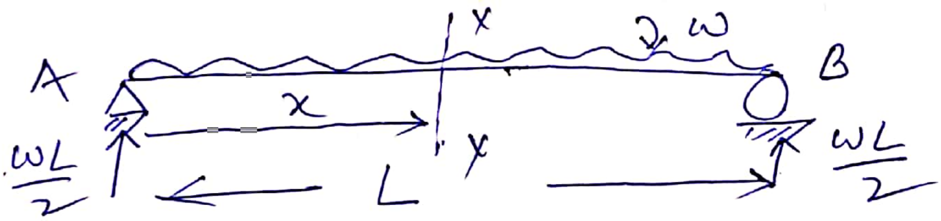
P-2.



$$S.E = \int_0^L \frac{M_x^2 \cdot dx}{2EI}$$

$$M_x = \left(\frac{wL}{2}\cdot x - \frac{wx^2}{2}\right)$$

$$S.E = \int_0^L \frac{1}{2EI}\left(\frac{wL}{2}\cdot x - \frac{wx^2}{2}\right)^2 dx$$

$$= \frac{1}{2EI}\int_0^L \left(\frac{w^2L^2}{4}\cdot x^2 + \frac{w^2x^4}{4} - \frac{2w^2L\cdot x^3}{4}\right) dx$$

$$= \frac{1}{2EI}\left[\frac{w^2L^2}{4}\left[\frac{x^3}{3}\right]_0^L + \frac{w^2}{4}\left[\frac{x^5}{5}\right]_0^L - \frac{w^2L}{2}\left[\frac{x^4}{4}\right]_0^L\right]$$

$$= \frac{1}{2EI}\left[\frac{w^2L^5}{12} + \frac{w^2L^5}{20} - \frac{w^2L^5}{8}\right]$$

$$= \frac{1}{2EI}\left[\frac{10w^2L^5 + 6w^2L^5 - 15w^2L^5}{120}\right]$$

$$= \frac{1}{2EI}\,\frac{w^2L^5}{120}$$

$$\boxed{SE = \frac{w^2L^5}{240\,EI}}$$

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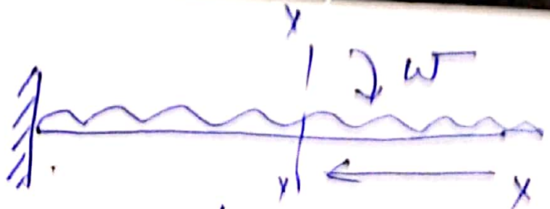
P-3

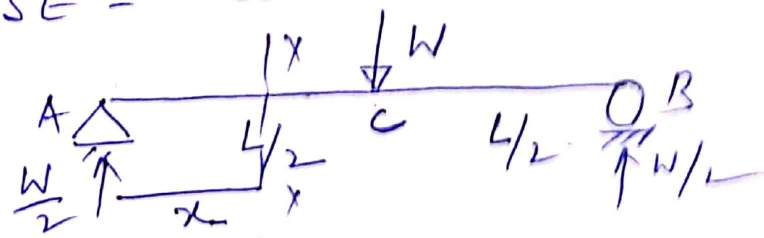


$$SE = \int_0^L \frac{M_x^2\,dx}{2EI} \quad \text{—— (1)}$$

$$M_x = \frac{-wx^2}{2}$$

$$SE =$$

P-4



$$SE = \int_0^L \frac{M_x^2\,dx}{2EI}$$

$$= \int_0^{L/2} \frac{M_x^2\,dx}{2EI} + \int_0^{L/2} \frac{M_x^2\,dx}{2EI}$$

$$= U_{AC} + U_{BC}$$

$$= 2\int_0^{L/2} \frac{M_x^2\,dx}{2EI}$$

$$M_x = \frac{W}{2}\cdot x$$

$$SE = 2\int_0^{L/2} \frac{1}{2EI}\cdot\left(\frac{W}{2}\cdot x\right)^2 dx$$

$$= \frac{1}{EI}\int_0^{L/2} \frac{W^2}{4}\cdot x^2\,dx$$

$$= \frac{W^2}{4EI}\left[\frac{x^3}{3}\right]_0^{L/2} = \frac{W^2}{12EI}\left[\frac{L^3}{8}\right]$$

$$SE = \frac{W^2L^3}{96EI}.$$

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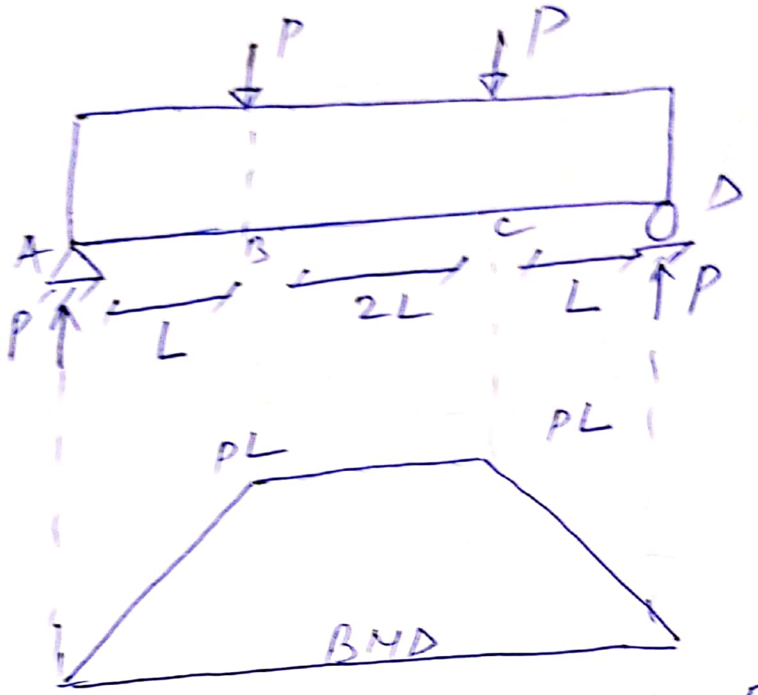
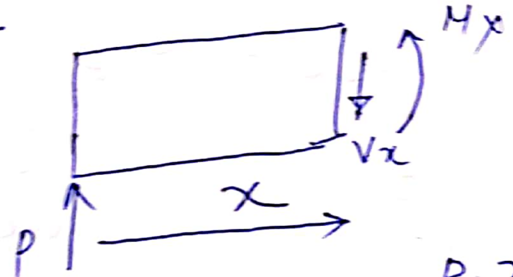
P-5 A simply supported beam in which load is applied as shown in the figure. Findout the total strain energy stored in a member.



Let U be the total strain Energy stored

$$U = U_{AB} + U_{BC} + U_{CD}$$

For AB



$$M_x = P.x$$

$$U_{AB} = \int_0^L \frac{M_x^2 dx}{2EI} = \int_0^L \frac{(P.x)^2 dx}{2EI}$$

$$U_{AB} = \frac{P^2L^3}{6EI}$$

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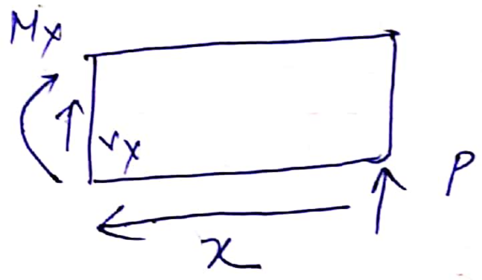
For BC

$$U_{BC} = \int_L^{3L} \frac{M_x^2\,dx}{2EI} = \int_L^{3L} \frac{(PL)^2 \cdot dx}{2EI}$$

$$= \frac{P^2L^3}{EI}$$

For member CD



$$-P \cdot x + M_x = 0$$

$$M_x = P \cdot x$$

$$U_{CD} = \int_0^L \frac{M_x^2 \cdot dx}{2EI} = \int_0^L \frac{(P \cdot x)^2 \cdot dx}{2EI}$$

$$= \frac{P^2L^3}{6EI}$$

$$U_{total} = U_{AB} + U_{BC} + U_{CD}$$

$$= \frac{P^2L^3}{6EI} + \frac{P^2L^3}{EI} + \frac{P^2L^3}{6EI}$$

$$\boxed{U_{total} = \frac{4P^2L^3}{3EI}}$$

(Happy Learning)

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