Department of Civil Engineering

Katihar Engineering College, Katihar

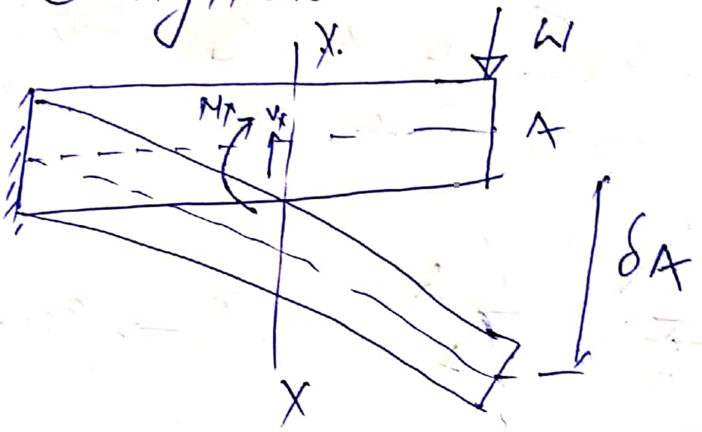
Subject: Introduction to Solid Mechanics

Topic : Slope and Deflection

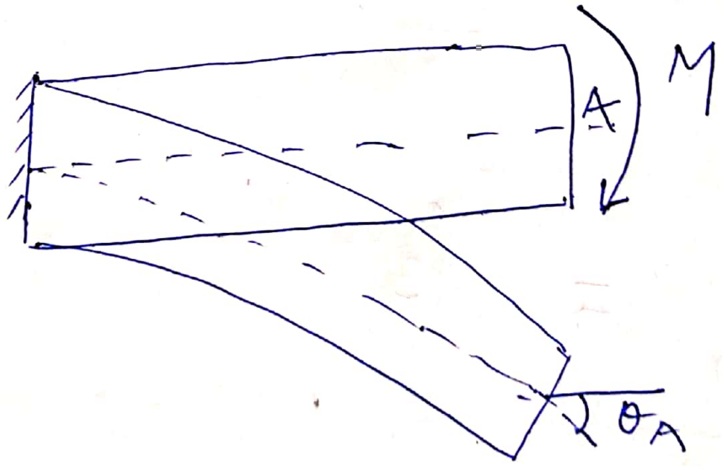
Lecture : 05

Instructor: Prof. RASHID MUSTAFA

⇒ Castigliano's Theorem.



$$\boxed{\frac{\partial U}{\partial W} = \delta_A}$$



$$\boxed{\frac{\partial U}{\partial M} = \theta_A}$$

$$U = \int_0^L \frac{M_x^2\,dx}{2EI}$$

$$\delta_A = \frac{\partial U}{\partial W} = \frac{\partial}{\partial W}\int_0^L \frac{M_x^2\,dx}{2EI}$$

$$\boxed{\delta_A = \int_0^L \frac{1}{2EI}(2M_x)\cdot\left(\frac{\partial M_x}{\partial W}\right)dx}$$

By this theorem deflection at any point where load is acting will be the partial derivative of total strain energy with respect to that load itself.

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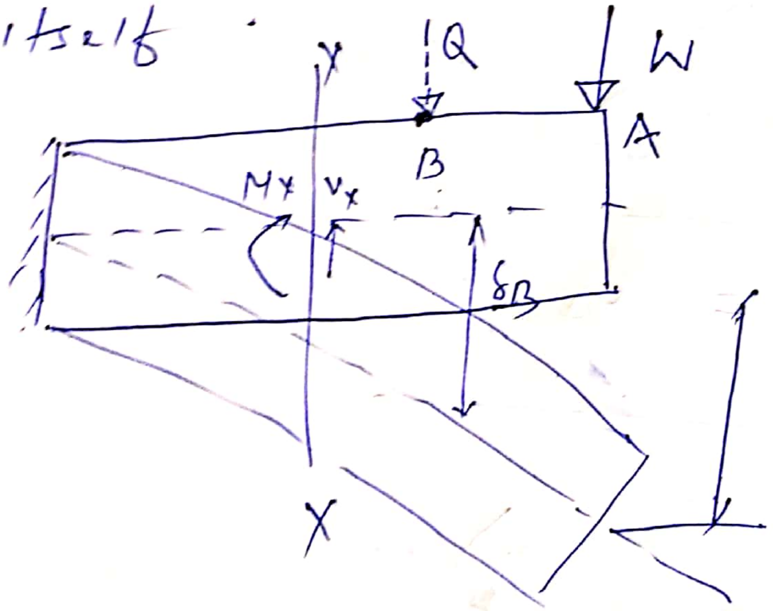
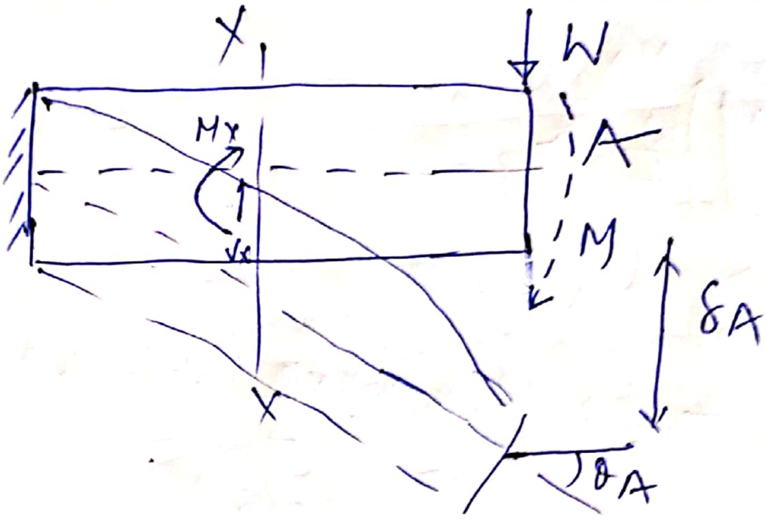


$Q \rightarrow$ Pseudo load

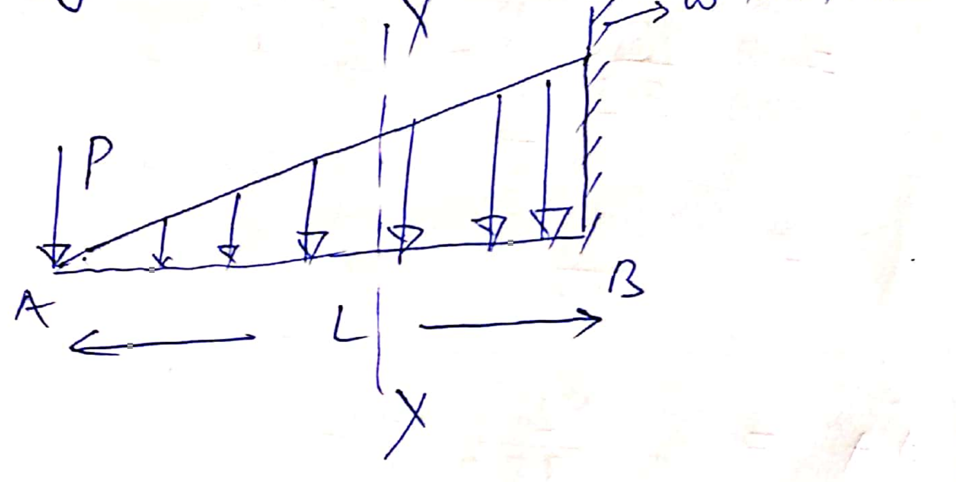
$$\boxed{\left.\frac{\partial U}{\partial Q}\right|_{Q=0} = \delta_B}$$

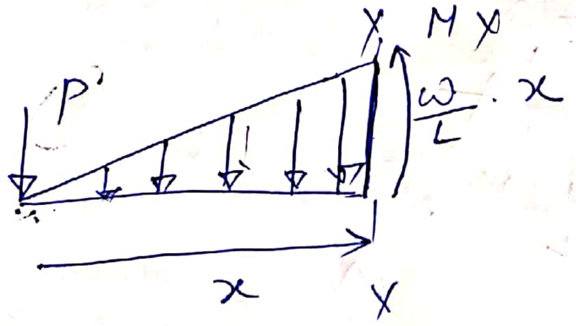


Where $M \rightarrow$ Pseudo Moment applied at point A

$$\boxed{\left.\frac{\partial U}{\partial M}\right|_{M=0} = \theta_A}$$

P-1. Determine the deflection at the free end of beam of constant x-section of length L in which concentrated load P is applied at the free end & uniformly varying load is applied full span of the beam. Also determine the slope at the free end by using strain Energy method.





$$-P.x - \frac{1}{2} \times \frac{\omega}{L}.x \times x \times \frac{x}{3} + (-M_x) = 0$$

$$M_x = -P.x - \frac{\omega x^3}{6L}.$$

$$U = \int_0^L \frac{M_x^2\,dx}{2EI}$$

By theorem deflection at the free end (P↓)

$$\frac{\partial U}{\partial P} = \delta_A = \frac{\partial}{\partial P}\int_0^L \frac{M_x^2\,dx}{2EI}$$

$$= \int_0^L \frac{1}{2EI}\cdot(2M_x)\cdot\left(\frac{\partial M_x}{\partial P}\right)dx$$

$$= \frac{1}{EI}\int_0^L \left(-P.x - \frac{\omega x^3}{6L}\right)\times(-x)\,dx$$

$$= \frac{1}{EI}\int_0^L \left(Px^2 + \frac{\omega x^4}{6L}\right)dx$$

$$= \frac{1}{EI}\left[\frac{Px^3}{3} + \frac{\omega}{6L}\cdot\frac{x^5}{5}\right]_0^L$$

$$= \frac{1}{EI}\left[\frac{PL^3}{3} + \frac{\omega}{30L}\cdot L^5\right]$$

$$\boxed{\delta_A = \frac{PL^3}{3EI} + \frac{1}{30}\frac{\omega L^4}{EI}}$$

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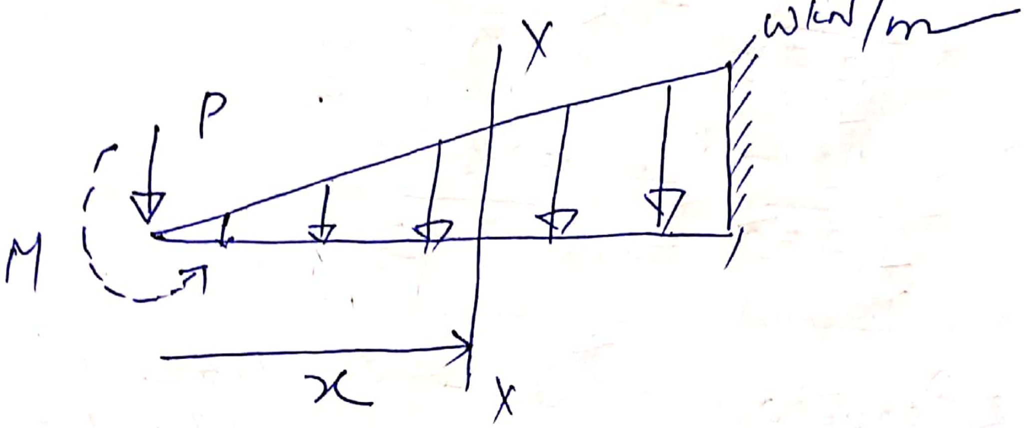
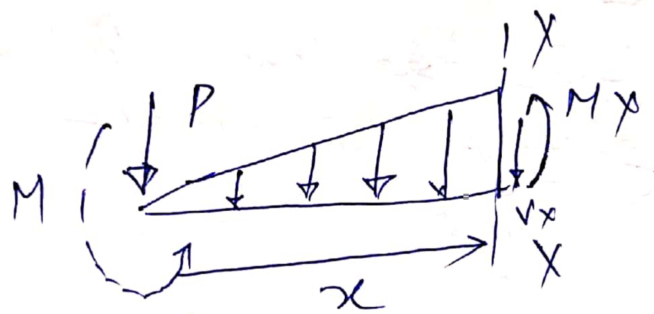
For Determination of Slope at point A (there is no moment at A, so we apply a pseudo moment at point A)





$$-P.x - M - \frac{1}{2} \times \frac{w}{L}.x.x.\frac{x}{3} - M_x = 0$$

$$M_x = -P.x - \frac{wx^3}{6L} - M$$

$$\theta_A = \frac{\partial U}{\partial M} \Big|_{M=0}$$

$$= \frac{\partial}{\partial M} \int_0^L \frac{M_x^2}{2EI}.dx$$

$$= \int_0^L \frac{1}{2EI}.2M_x.\left(\frac{\partial M_x}{\partial M}\right).dx$$

$$= \frac{1}{EI} \int_0^L M_x(-1)\,dx$$

$$= \frac{1}{EI} \int_0^L \left(P.x + \frac{wx^3}{6L} + M\right) dx$$

$$= \frac{1}{EI}\left[\frac{PL^2}{2} + ML + \frac{1}{6}\frac{wL^4}{4L}\right]_{M=0}$$

$$\theta_A = \frac{PL^2}{2EI} + \frac{wL^3}{24EI}$$

$$\boxed{\theta_A = \frac{PL^2}{2EI} + \frac{wL^3}{24EI}}$$

NOTE: Pseudo moment direction is along the beam deflection. If use opposite direction negative sign multiply for getting the exact result.

<Happy Learning>