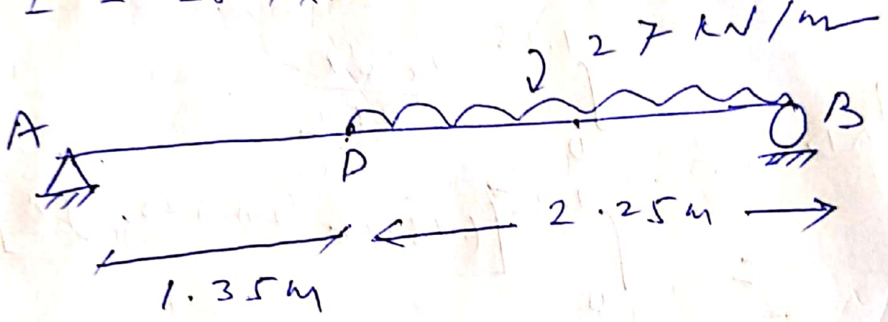
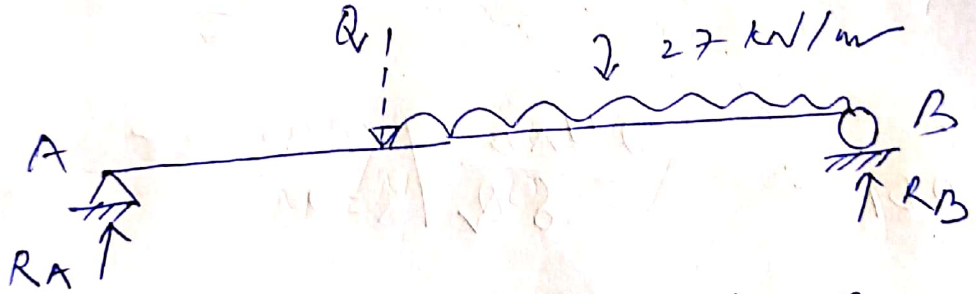
Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Slope and Deflection
Lecture: 06
Instructor: Prof. RASHID MUSTAFA

P-2 A simply supported beam as shown in the figure in which uniformly distributed load is acting. Find out the deflection at point D by using strain energy method

$I = 28.9 \times 10^{6}\ mm^4$.





$$R_A + R_B = (27 \times 2.25) + Q$$

$$R_A + R_B = 60.75 + Q \qquad \text{--- (1)}$$

$$\Sigma M_A = 0$$

$$-R_B \times 3.6 + \left(27 \times 2.25 \times \left(1.35 + \frac{2.25}{2}\right)\right) + Q \times 1.35 = 0$$

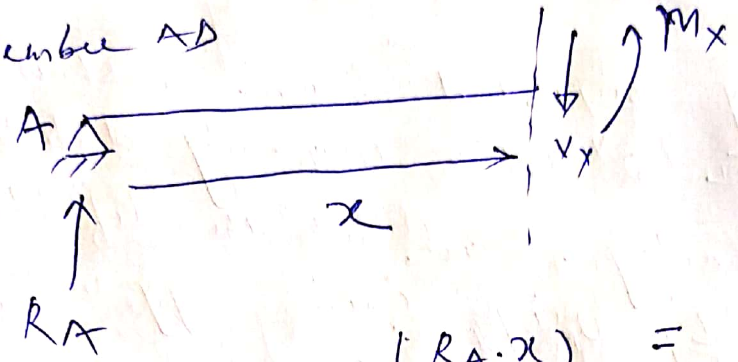
$$R_B \times 3.6 = 150.36 + 1.35Q$$

$$R_B = 41.77 + 0.375Q \quad \text{--- (2)}$$

$$R_A = 60.75 + Q - 41.77 - 0.375Q$$

$$R_A = (18.98 + 0.625Q)$$

For Member AD



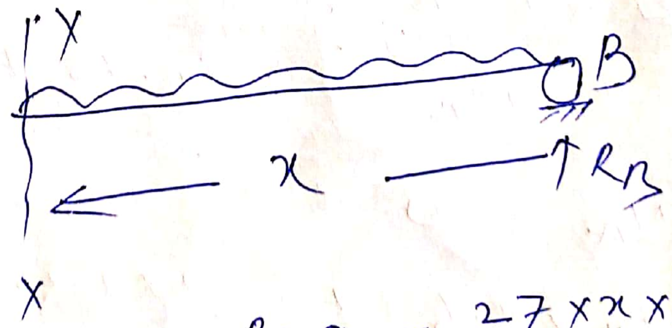
$$M_x = (R_A \cdot x) = (18.98 + 0.625Q) \cdot x$$

$$U_{AD} = \int_0^{1.35} \frac{M_x^2 \, dx}{2EI}$$

$$= \int_0^{1.35} \frac{[(18.98 + 0.625Q)x]^2 \, dx}{2EI}$$

$$\delta_{D1} = \frac{\partial U_{AD}}{\delta Q}\bigg|_{Q=0}$$

For Member BD



$$M_x = R_B \cdot x - 27 \times x \times \frac{x}{2}$$

$$= (R_B \cdot x - 13.5x^2)$$

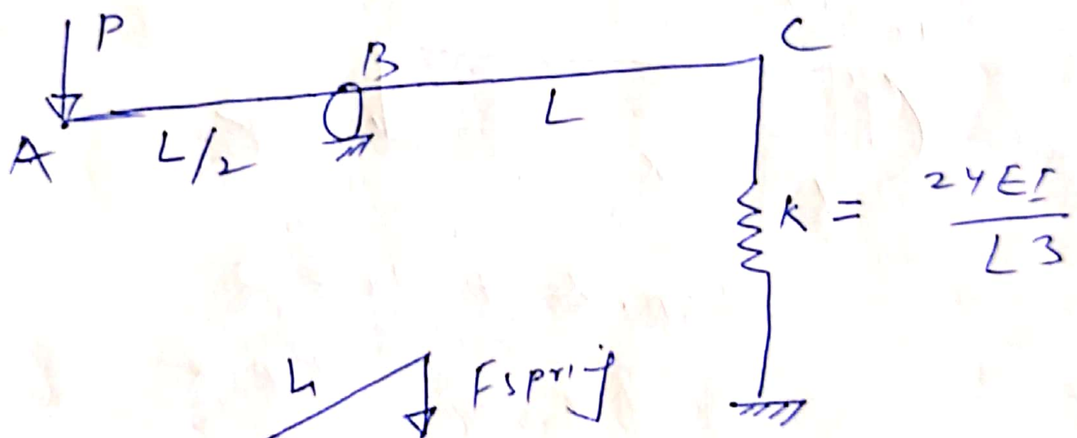
$$U_{BD} = \int_0^{2.25} \frac{M_x^2\,dx}{2EI}$$

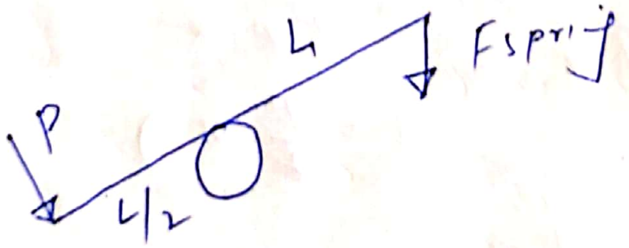
$$\delta_{D_2} = \left.\frac{\partial U_{BD}}{\partial Q}\right|_{Q=0}$$

Total deflection at Point D

$$\boxed{\delta_D = \delta_{D_1} + \delta_{D_2}}$$

Q-3. Determine the deflection at point A of the beam as shown in figure in which spring stiffness K spring is attached of spring by using strain energy method





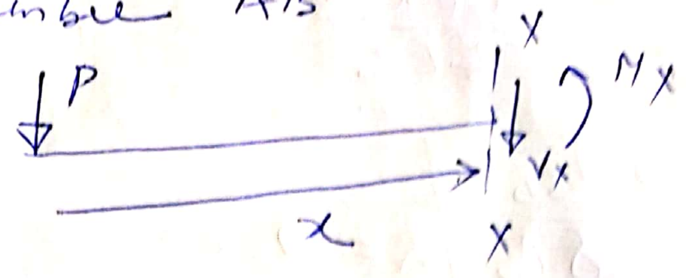
$$P \times \frac{L}{2} = F_{spring} \times L$$

$$\boxed{F_{spring} = \frac{P}{2}}$$

Let U_T be the total strain energy.

$$U_T = U_{AB} + U_{BC} + U_{spring}$$

For member AB



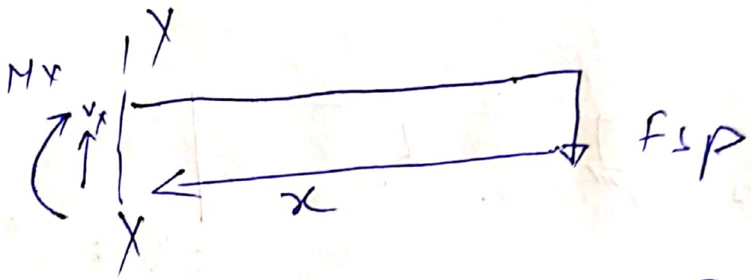
$$-P \cdot x + (-M_x) = 0$$

$$M_x = -P \cdot x$$

$$U_{AB} = \int_0^{L/2} \frac{M_x^2 \, dx}{2EI}$$

$$\delta_{A_1} = \frac{\partial U_{AB}}{\delta P}$$

For Member BC



$$F_{sp} \cdot x + M_x = 0$$

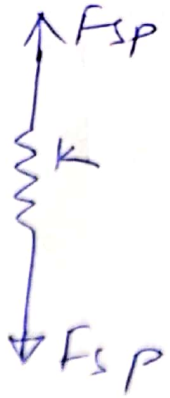
$$M_x = -F_{sp} \cdot x$$

$$= \left(-\frac{P}{2} \cdot x\right)$$

$$U_{BC} = \int_0^{L} \frac{M_x^2 \, dx}{2EI}$$

$$\delta_{A_2} = \frac{\delta U_{BC}}{\delta P}$$

For spring



$$\text{Strain energy } (U_{spring}) = \frac{F_{sp}^2}{2k}$$

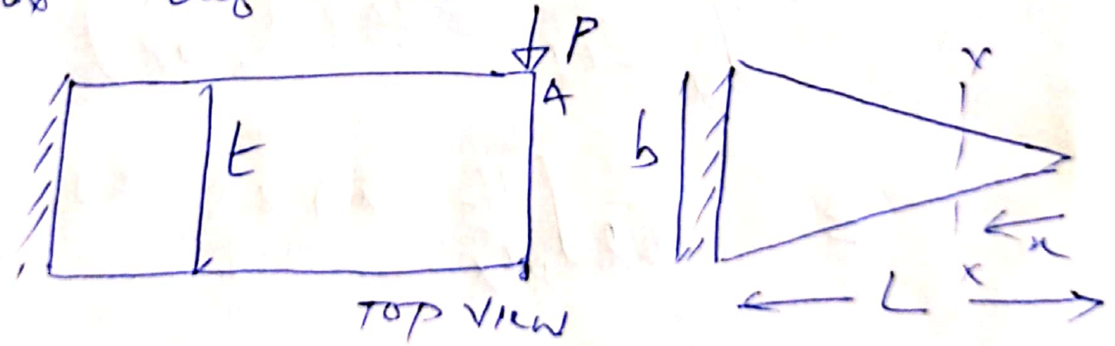
$$U_{spring} = \frac{(P/2)^2}{2k}$$

$$U_{spring} = \frac{P^2}{8k}$$

$$\delta_{A_3} = \frac{\partial U_{spring}}{\delta P}$$

$$\text{Total deflection } (\delta_A) = \delta_{A_1} + \delta_{A_2} + \delta_{A_3}$$

P-3. A triangular shape cantilever beam of uniform thickness as shown in the fig in which a concentrated load is applied at the free end. Determine the maxm deflection at the free end.

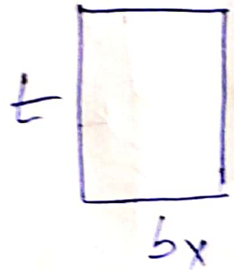


TOP VIEW

Since width is varying

at a distance x

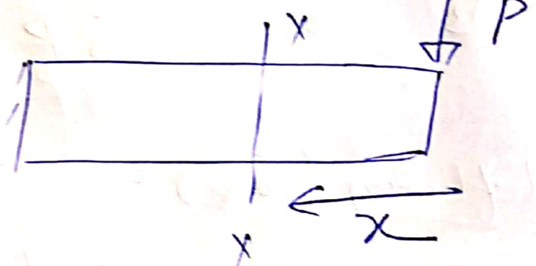
$$\text{width} = \left(\frac{b}{L}\right) \cdot x$$



$$I_x = \frac{b_x t^3}{12} = \frac{\left(\frac{b}{L}\cdot x\right) t^3}{12}$$

$$I_x = \frac{b \cdot x\, t^3}{12L}$$

$$\delta_A = \delta_{max} = \frac{\partial U}{\partial P}$$



$$M_x = -P \cdot x$$

$$\delta_A = \delta_{max} = \frac{\partial U}{\partial P} = \frac{\partial}{\partial P}\int_0^L \frac{M_x^2\, dx}{2EI}$$

$$= \int_0^L \frac{1}{2EI}(2M_x)\cdot\left(\frac{\partial M_x}{\partial P}\right) dx$$

$$= \frac{1}{E}\int_0^L \frac{1}{I_x}(-P\cdot x)\times(-x)\; dx$$

$$= \frac{1}{E}\int_0^L \frac{Px^2 \times 12L}{b x t^3}\, dx$$

$$= \frac{1}{E}\int_0^L \frac{12PL\cdot x}{bt^3}\, dx$$

$$\delta_{max} = \frac{12LP}{Ebt^3}\left[\frac{x^2}{2}\right]_0^L = \frac{6PL^3}{Ebt^3}$$