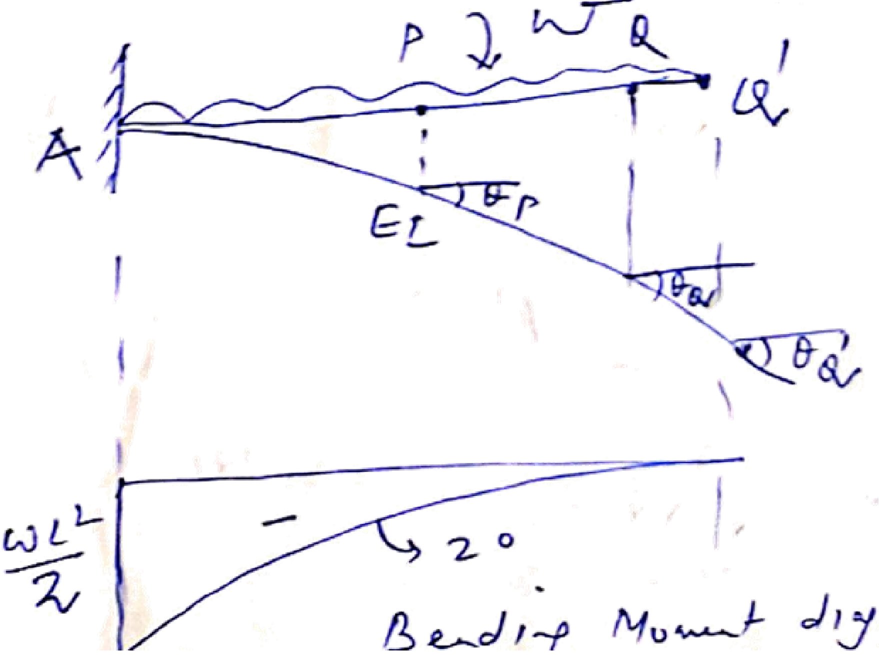
Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Slope and Deflection (Moment Area Method)
Lecture: 07
Course Instructor: Prof. Rashid Mustafa

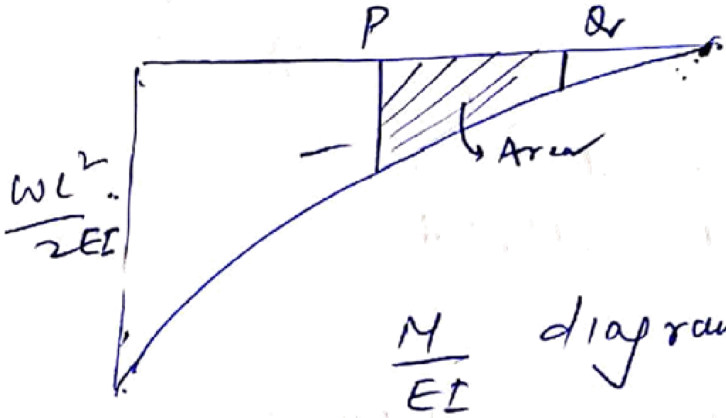
⇒ Moment Area Method (Mohr's Method)

- This method is sutable when it is easy to findout area of bending moment diagram and Centroid of that Area
- It is not sutable for those beam in which internal hinges are present where Method is not sutable where
- Slope changes suddenly.
- This method is also used for non-Prismatic Section

⇒ Mohr's Ist theorem



Bending Moment diagram (M)

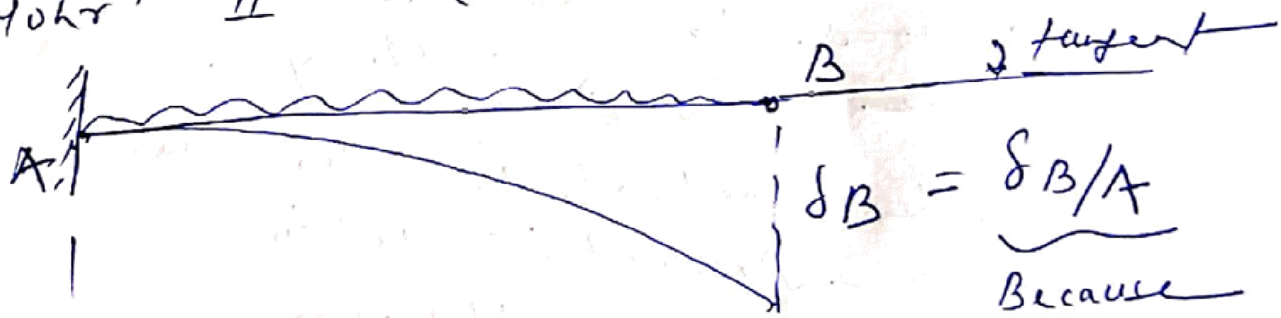


$\frac{M}{EI}$ diagram

$$\boxed{\theta_Q - \theta_P = \text{Area of } \frac{M}{EI} \text{ diagram}}$$ → Mohr's Ist theorem

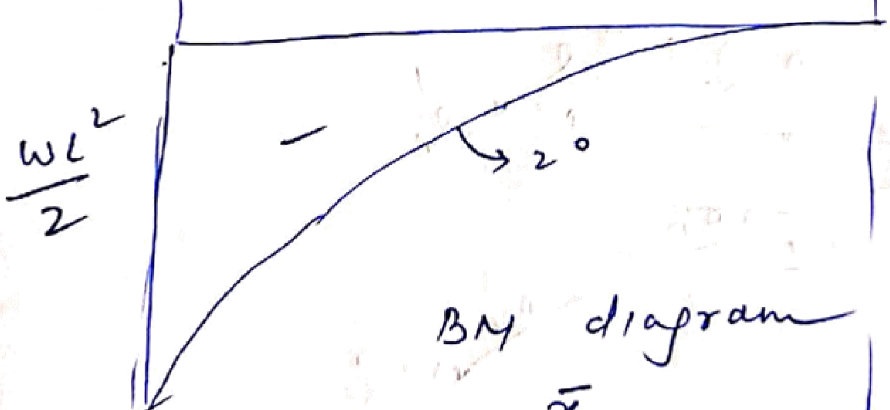
By Mohr's Ist theorem change of slope from any point P to Q is equal to area of $\frac{M}{EI}$ diagram b/w P and Q.

⇒ Mohr's IInd theorem

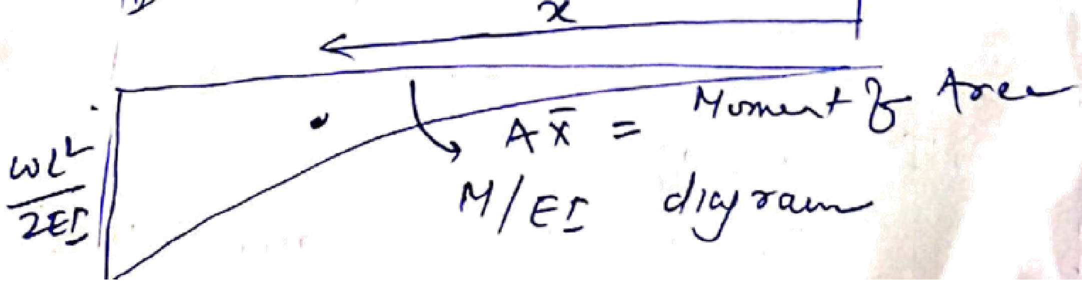


$\delta_B = \delta_{B/A}$

Because tangent is parallel to x-axis



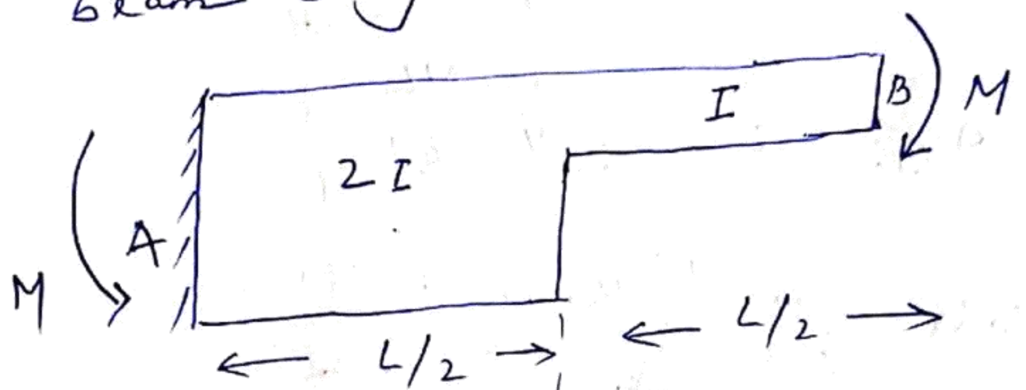
BM diagram

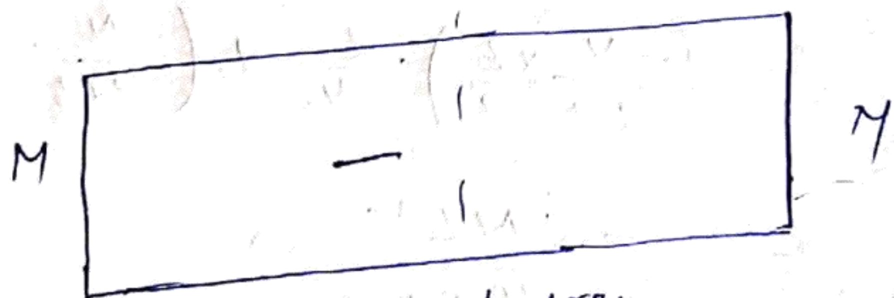


M/EI diagram

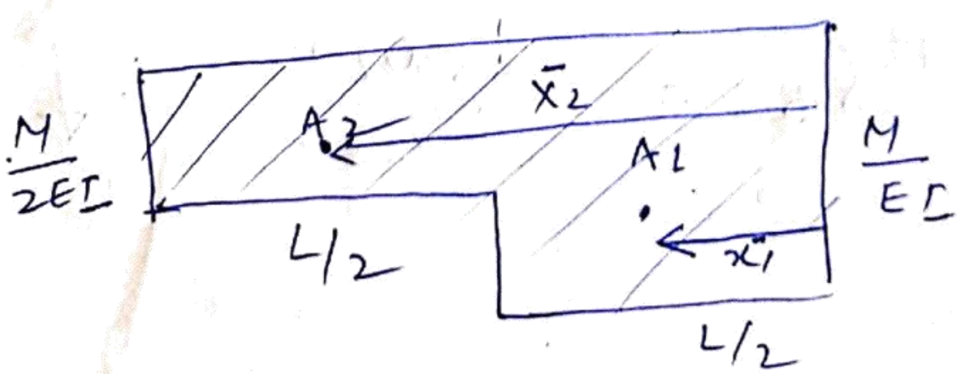
$\boxed{\delta_{B/A} = \text{Moment of area of } \frac{M}{EI} \text{ diagram b/w A and B about B}}$

P-1. A non prismatic beam of length L as shown in the figure. Findout the slope and deflection at the free end B of the beam using Moment Area method.





BM diagram



$\frac{M}{EI}$ diagram

A/c to Mohr's Ist theorem

$$\theta_B - \theta_A = \text{Area of } \frac{M}{EI} \text{ digram}$$

$$\theta_B - \theta_A = A_1 + A_2$$

$$= \left(-\frac{M}{EI} \times \frac{L}{2}\right) + \left(-\frac{M}{2EI} \times \frac{L}{2}\right)$$

$$\theta_B - 0 = -\frac{3}{4}\frac{ML}{EI}$$

$$\boxed{\theta_B = -\frac{3}{4}\frac{ML}{EI}}$$

A/c to Mohr's -II nd theorem

$$\delta_{B/A} = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$= \left(\frac{-M}{EI} \times \frac{L}{2}\right) \cdot \frac{L}{4} + \left(\frac{-M}{2EI} \cdot \frac{L}{2}\right) \times \frac{3L}{4}$$

$$\delta_{B/A} = \frac{-5ML^2}{16EI}$$

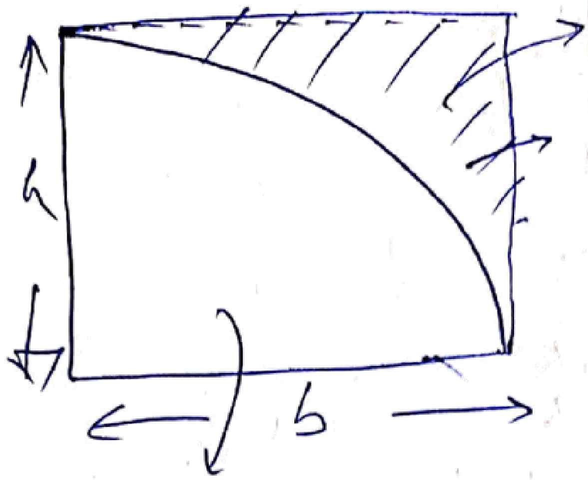
$$\delta_B - \delta_A = \frac{-5ML^2}{16EI}$$

$$\boxed{\delta_B = \frac{-5ML^2}{16EI}}$$ $\downarrow$ (downward deflection)

⇒ Properties of Area.

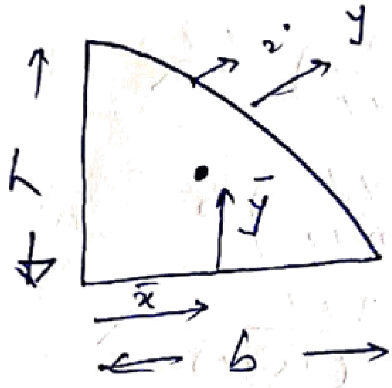
smaller Area

parabolic spandral



Larger Area (semi segment)

$$y = h\left(1-\frac{x^2}{b^2}\right)$$



$$\boxed{A = \frac{2}{3} bh}$$

$$\bar{x} = \frac{3}{8} b$$

$$\bar{y} = \frac{2}{5} h$$

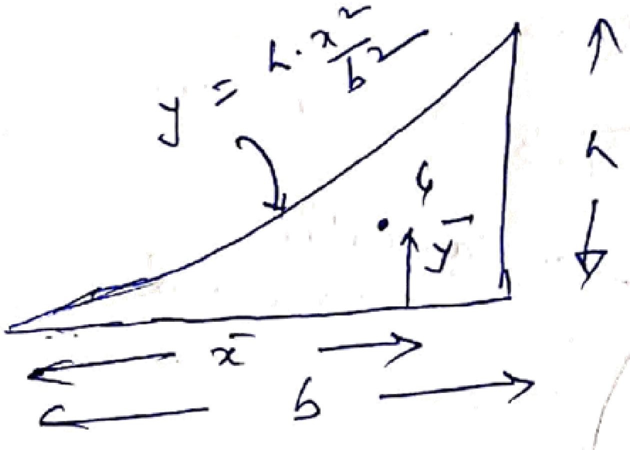
For n^{th} degree.

$$A = \left(\frac{n}{n+1}\right) b \cdot h$$

$$\bar{x} = \frac{n+1}{2(n+2)} \cdot b$$

$$\bar{y} = \left(\frac{n}{2n+1}\right) \cdot h$$

⇒



$$A = \frac{bh}{3}$$

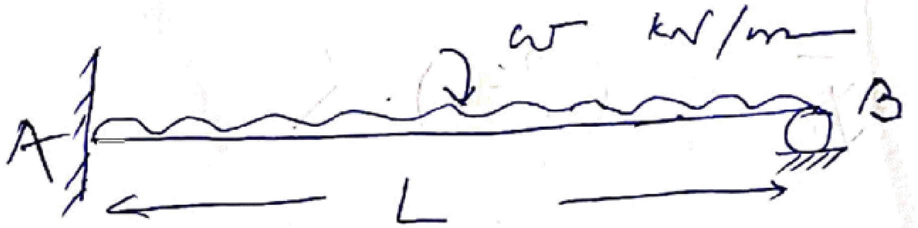
$$\bar{x} = \frac{3}{4} b$$

$$\bar{y} = \frac{3}{10} h .$$

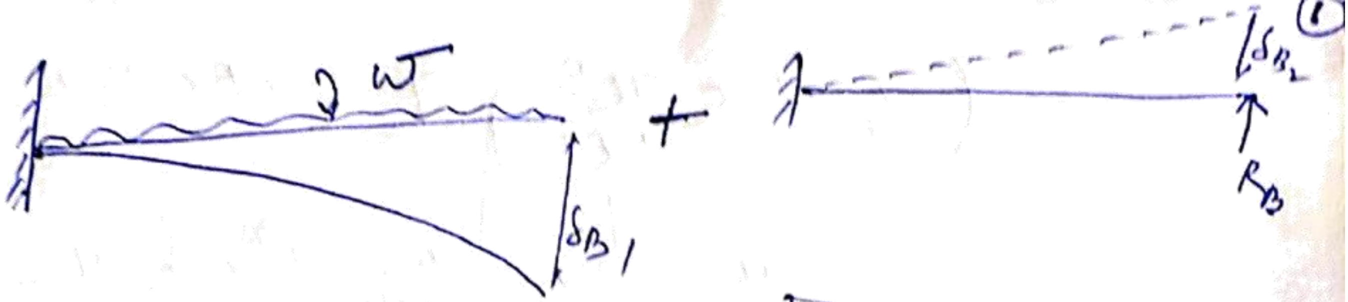
For n^{th} degree

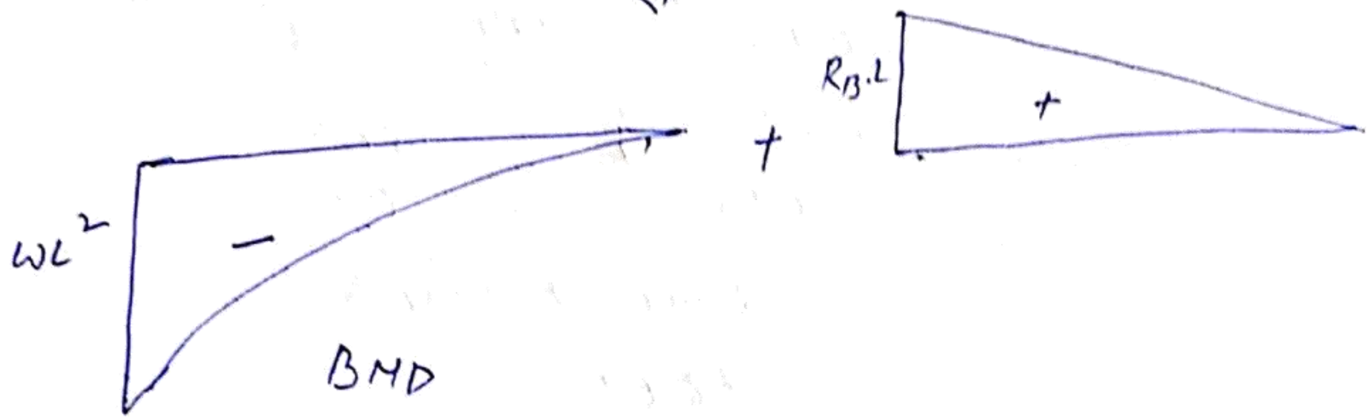
$$\boxed{\begin{aligned} A &= \left(\frac{1}{n+1}\right) bh \\ \bar{x} &= \left(\frac{n+1}{n+2}\right) b \\ \bar{y} &= \frac{n+1}{2(2n+1)} \cdot h \end{aligned}}$$

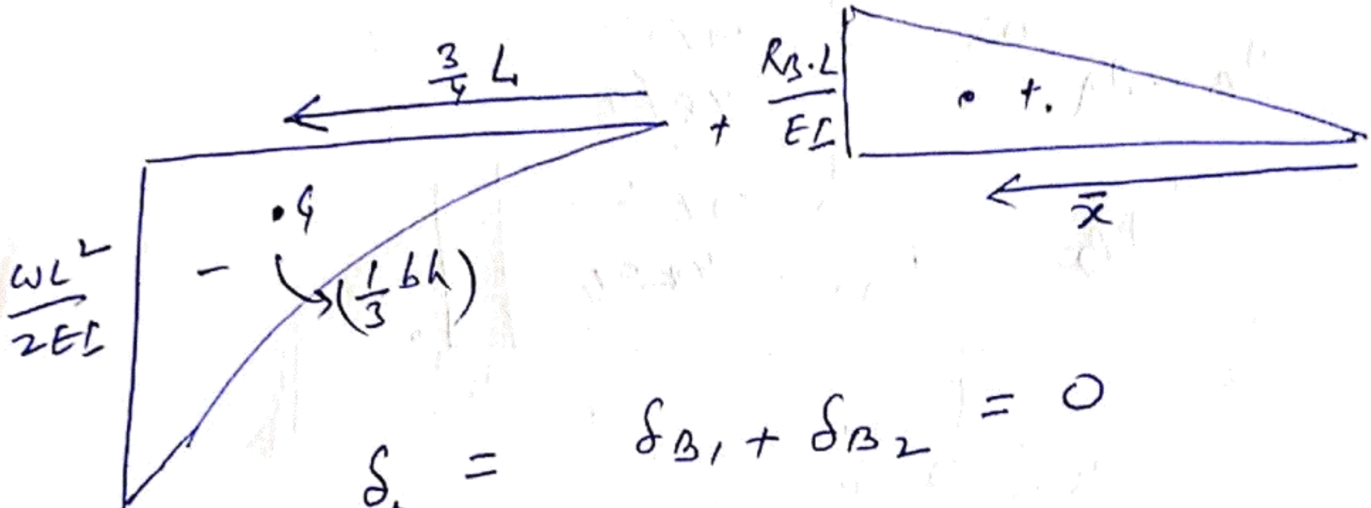
P-2. For the Propped Cantilever beam as shown in the figure, findout the propped reaction & slope at point B by using Area-moment method.



(1)







$$\delta_B = \delta_{B_1} + \delta_{B_2} = 0$$

$$\left(-\frac{1}{3} \times L \times \frac{wL^2}{2EI} \times \frac{3}{4}L\right) + \left(\frac{1}{2} \times \frac{R_B L}{EI} \times L \times \frac{2L}{3}\right) = 0$$

$$\frac{-wL^4}{8EI} + \frac{R_B L^3}{3EI} = 0$$

$$\boxed{R_B = \frac{3}{8}wL}$$

A/c to Mohr's Ist theorem

$$\theta_B - \theta_A = \theta_{B_1} + \theta_{B_1}$$

Scanned with CamScanner

$$= \left(-\frac{1}{3} \times L \times \frac{wL^2}{2EI}\right) + \frac{1}{2} \times L \times \frac{R_B \cdot L}{EI}$$

$$= \frac{-wL^3}{6EI} + \frac{1}{2EI} \times L^2 \times \frac{3}{8} wL$$

$$= \frac{-wL^3}{6EI} + \frac{3}{16} \frac{wL^3}{EI}$$

$$= \frac{-8wL^3 + 9wL^3}{48EI}$$

$$\theta_B - \cancel{\theta_A}^{\,0} = \frac{wL^3}{48EI}$$

$$\boxed{\theta_B = \frac{wL^3}{48EI}}$$