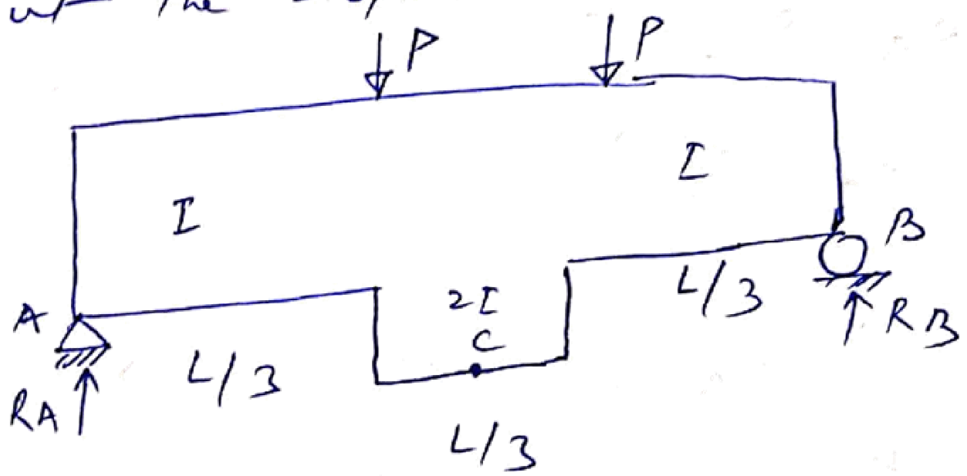
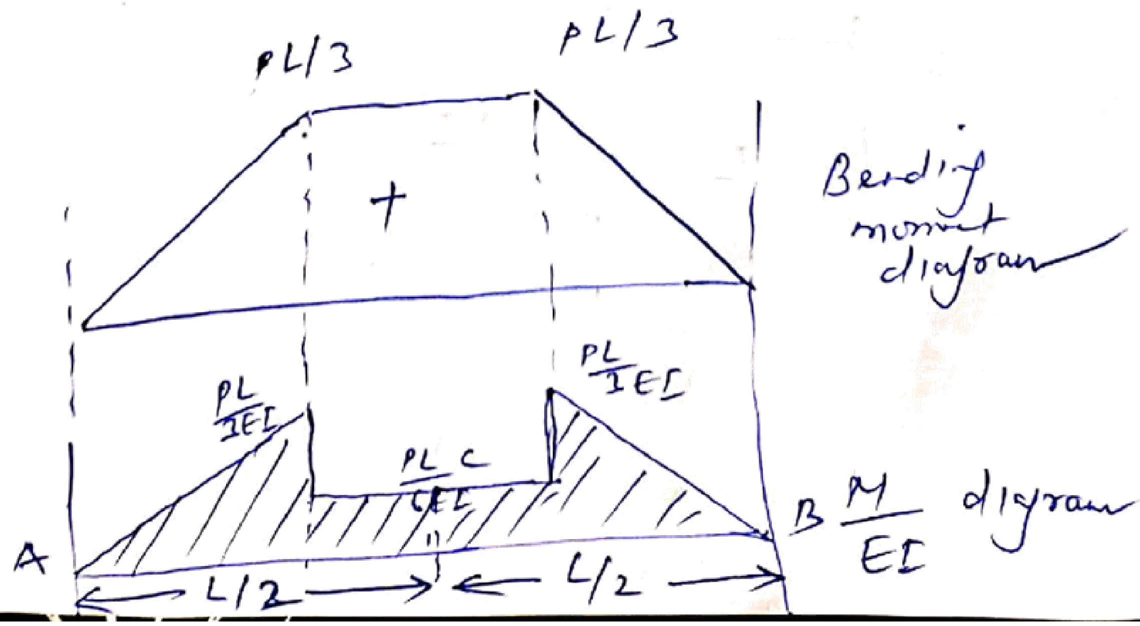
Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Slope and Deflection (Conjugate Beam Method)
Lecture: 08
Course Instructor: Prof. Rashid Mustafa

P-3. A non-Prismatic beam of length L in which load is applied as shown in the figure. Find out the slope at point A and B.



$$R_A + R_B = 2P$$
$$R_A = R_B = P$$



$$\theta_C - \theta_A = \left(\frac{1}{2} \times \frac{PL}{3EI} \times \frac{L}{3}\right) + \frac{PL}{6EI} \times \frac{L}{6}$$

$$\theta_C - \theta_A = \frac{PL^2}{18EI} + \frac{PL^2}{36EI}$$

$$0 - \theta_A = \frac{3PL^2}{36EI} = \frac{PL^2}{12EI}$$

$$\boxed{\theta_A = -\frac{PL^2}{12EI}}$$

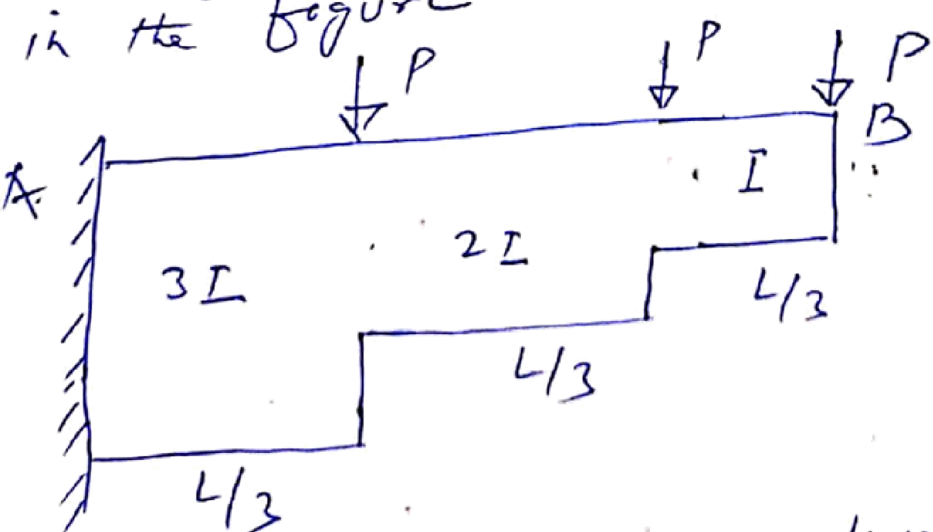
$$\theta_B - \theta_C = \left(\frac{1}{2} \times \frac{PL}{3EI} \times \frac{L}{3}\right) + \frac{PL}{6EI} \times \frac{L}{6}$$

$$\theta_B - \cancel{\theta_C}^{\,0} = \frac{PL^2}{18EI} + \frac{PL^2}{36EI}$$

$$\boxed{\theta_B = \frac{PL^2}{12EI}}$$

Home Assignment

P-4: Find out the slope and deflection at the free end of the cantilever as shown in the figure



$$\begin{cases} \theta_B = \dfrac{PL^2}{3EI} \\ \delta_B = \dfrac{-23}{108}\dfrac{PL^3}{EI} \end{cases}$$

$\theta_B - \theta_A =$ Area of $\frac{M}{EI}$ diagram

$\theta_B - \theta_A =$ Area of $\frac{M}{EI}$ diagram

⇒ Conjugate Beam method :

- This method is applicable for Prismatic and non-Prismatic beam both.
- Conjugate beam is an imaginary beam for which loading diagram is $\frac{M}{EI}$ diagram for the given beam
- This method can be used to the beam with internal hinges.

Mohr's Ist theorem

Slope at any point for a given beam is equal to the shear force (V) at that point in the Conjugate beam.

$$(\theta)_{\text{Real beam}} = (V)_{\text{Conjugate beam}}$$

Mohr's II theorem

Deflection at any point in given beam is equal to the bending moment at that point in Conjugate beam.

$$(\delta)_{\text{Real beam}} = (M)_{\text{Conjugate beam}}$$

Note: If $\frac{M}{EI}$ diagram is positive (sagging) then loading in conjugate beam will be upward & vice versa

(2) If Shear force (V) in conjugate beam is positive then slope in given beam is +ve (Anticlockwise) & vice versa.

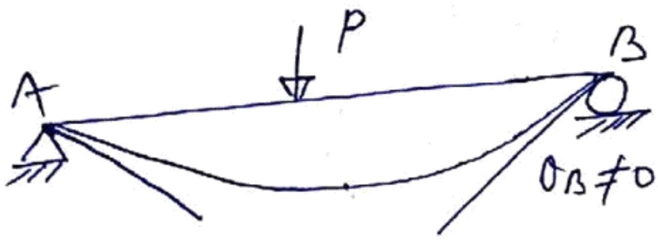
(3) If Bending moment in conjugate beam is +ve then deflection in given beam is +ve (upward) and vice versa.

⇒ Comparison b/w Real & Conjugate beam

Real Beam	Conjugate beam

1.

Real Beam:



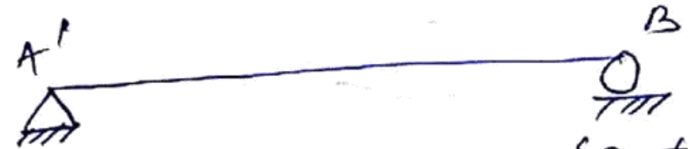
$\theta_A \neq 0$

$\delta_A = 0$

$\theta_B \neq 0$

$\delta_B = 0$

Conjugate beam:



$SF_A \neq 0$

$M_A = 0$

$SF_B \neq 0$

$M_B = 0$

OR

$SF_A \neq 0$

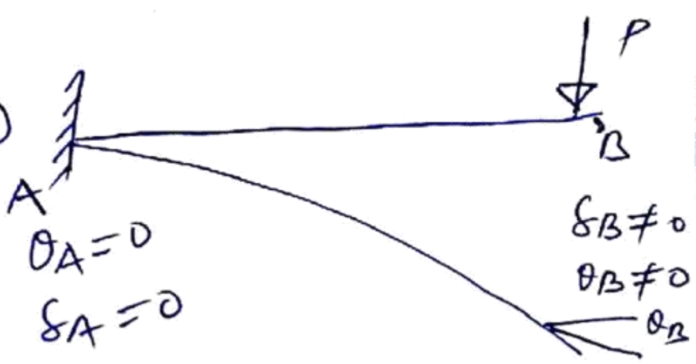
$M_A = 0$

$SF_B \neq 0$

$M_B = 0$

(2)

Real Beam:



$\theta_A = 0$

$\delta_A = 0$

$\delta_B \neq 0$

$\theta_B \neq 0$

Conjugate beam:

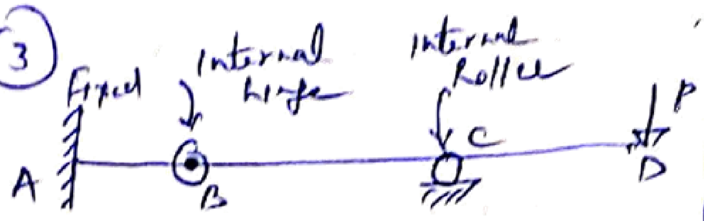
$SF_A = 0$

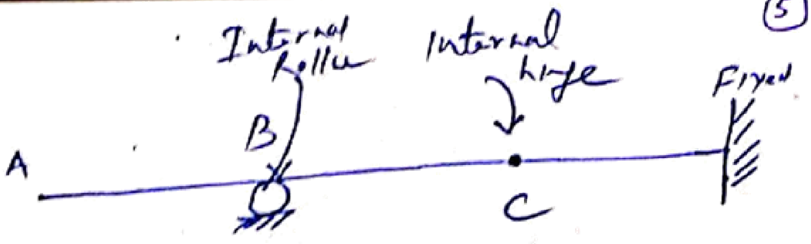
$M_A = 0$

$SF \neq 0$

$M \neq 0$

(3)





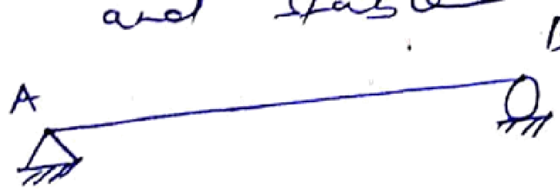
Fixed End → Free end

Free end → Fixed End

Internal hinge → Internal Roller

Internal Roller → Internal hinge

(4) If given beam is determinate & stable then Conjugate beam is also determinate and stable



Simply support (hinge support) → Roller support

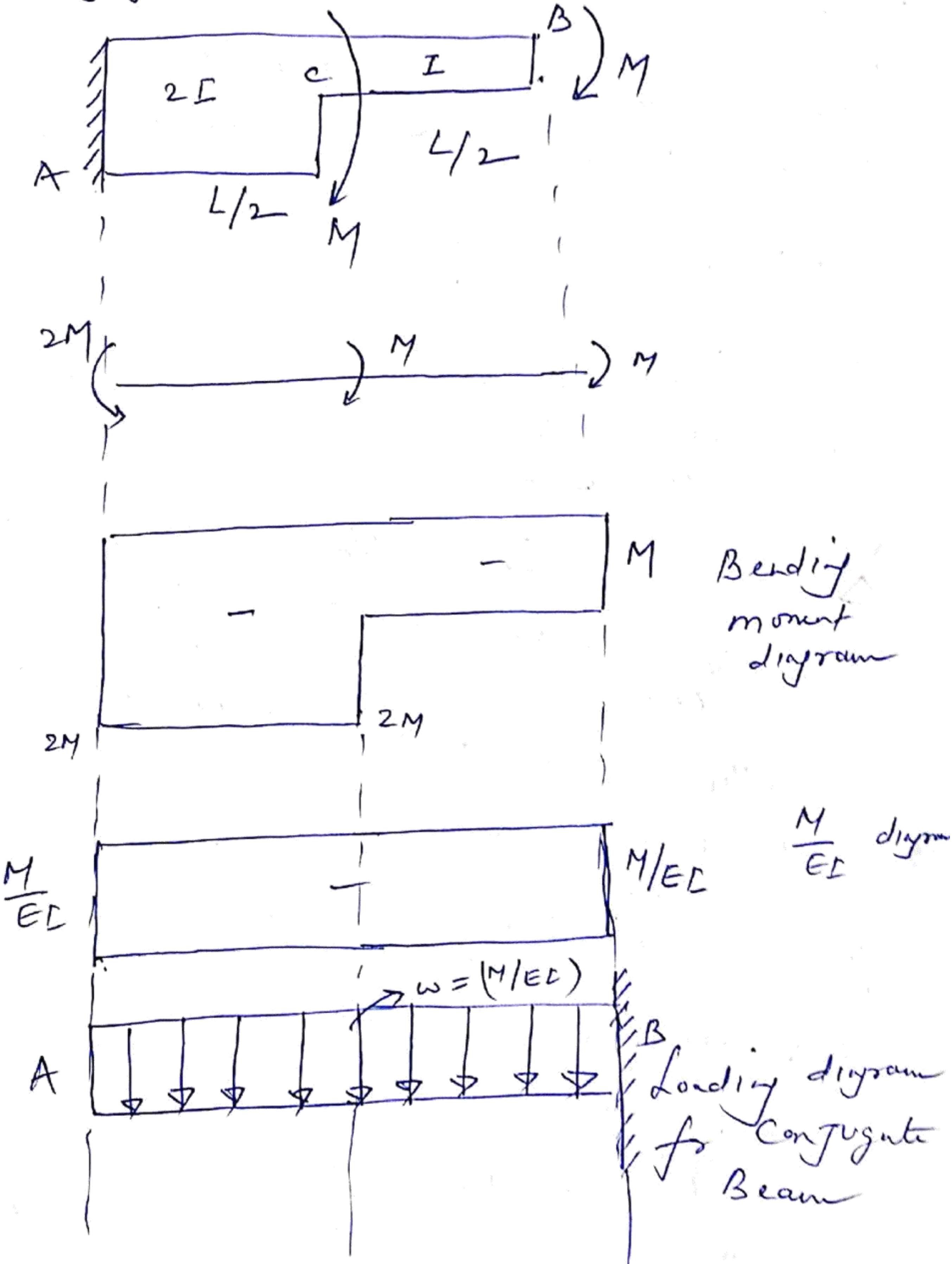
Roller Support → Simply support (hinge support)

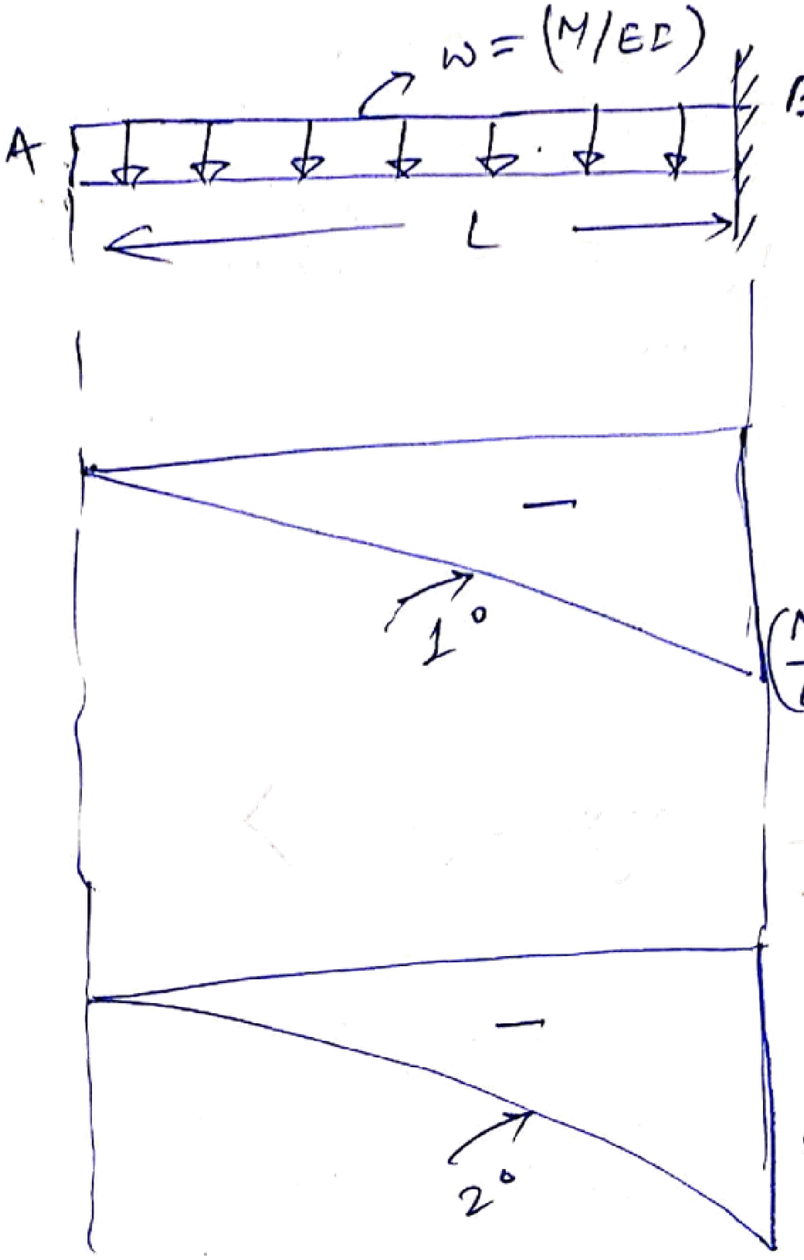
(5) If Given beam is indeterminate then Conjugate beam is unstable and if given beam is unstable then Conjugate beam is indeterminate.

Beam (Indeterminate) → Unstable

Beam (Unstable) → Indeterminate

P-1. For the beam with the loading as shown in the figure: Findout the slope and deflection at the free end by Conjugate beam method.





(Loading diagram for Conjugate beam)

(Shear force Diagram)

(Bending Moment diagram)

$\frac{wL^2}{2EI} = \left(\frac{ML^2}{2EI}\right)$

$(\text{Slope at point } B)_{\text{Real beam}} =$ Shear force at that point in Conjugate beam

$$(\theta_B)_{\text{given/Real beam}} = -\frac{ML}{EI}$$

$(\text{deflection at point } B)_{\text{Real beam}} =$ Bending moment at that point in Conjugate beam

$$(\delta_B)_{\text{Real beam}} = (BM)_{C \cdot B}$$

$$(\delta_B)_{\text{Real beam}} = -\frac{ML^2}{2EI}$$ (downward deflection)

HAPPY LEARNING