

Strongly connected component

↳ A directed graph is strongly connected if there is a path between all pairs of vertices. A strongly connected component (SCC) of a directed graph is a maximal strongly connected subgraph. For example, there are 3 SCCs in the following graph.



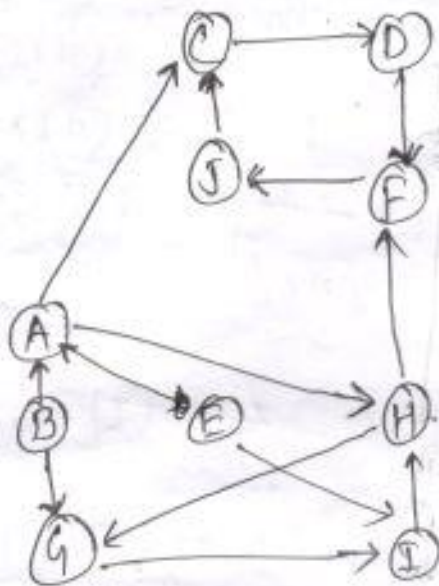
$$\text{SCC} = \{ \{1, 2, 5\}, \{3, 4\}, \{4\} \}$$

⇒ we can find all strongly connected components in $O(V+E)$ time using Kosaraju's algorithm.

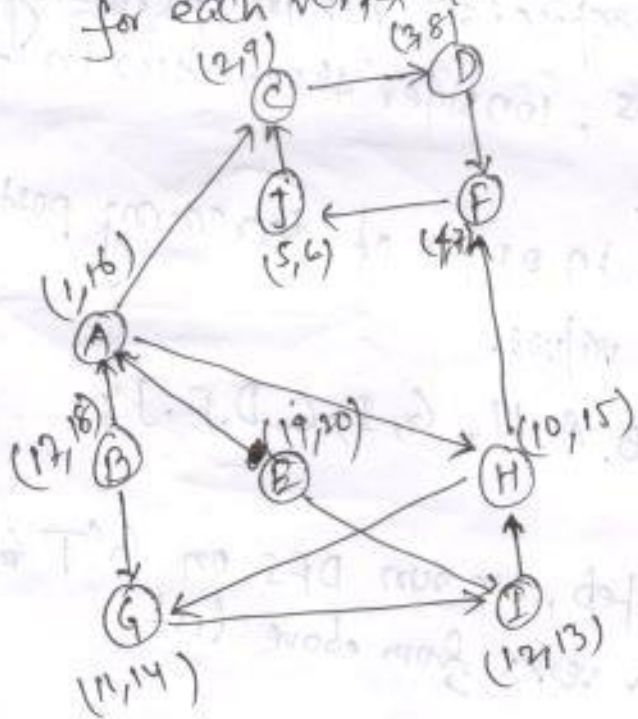
KOSARAJU ALGORITHM

- ① Call DFS(G) to compute finishing time $f(u)$ for each vertex u .
- ② Compute Transpose(G).
- ③ Call DFS(Transpose(G)), but in the main loop of consider the vertices in order of decreasing $f(u)$.
- ④ Output the vertices of each tree in depth-first forest of step 3 as a separate strong connected component.

Ex

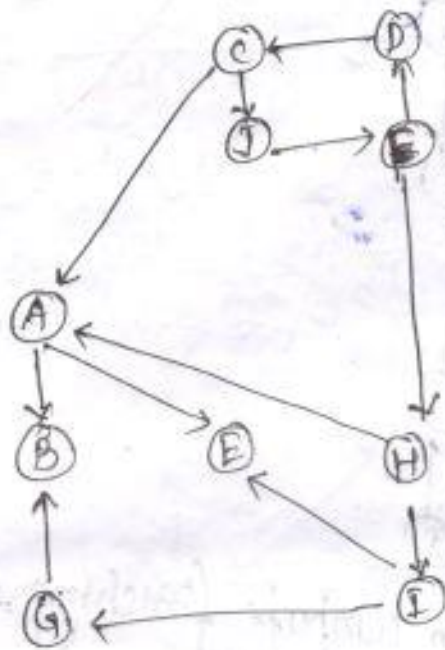


steps Call DFS(G) to compute finishing times $f(u)$ for each vertex u .



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next

Step 2 Compute the transpose (G^T)



Step 3 Call DFS (G^T), but in decreasing + main loop of DFS, consider the vertices in dec order of $F[u]$.

→ Okay, so vertices in order of decreasing post-visit (finishing times) values.

{ E, B, A, H, G, I, C, D, F, J }

→ So at this step, we run DFS on G^T but start with each vertex from above list.

$$\text{DFS}(E) = \{E\}$$

$$\text{DFS}(B) = \{B\}$$

$$\text{DFS}(A) = \{A\}$$

$$\text{DFS}(H) = \{H, I, G\}$$

$$\text{DFS}(G) = \{ \text{remove from list since it is already visited} \}$$

$$\text{DFS}(E) =$$

$$\text{DFS}(C) = \{C, J, F, D\}$$

$$\left. \begin{array}{l} \text{DFS}(J) \\ \text{DFS}(F) \\ \text{DFS}(D) \end{array} \right\} \text{removed from list as it is already visited.}$$

