The Bellman-Ford algorithm

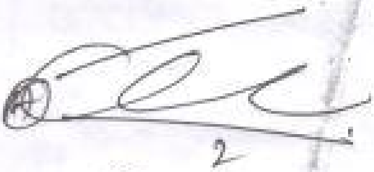
→ Dijkstra algorithm works only on non-negative weight where as Bellman-Ford can work with -ve weight.

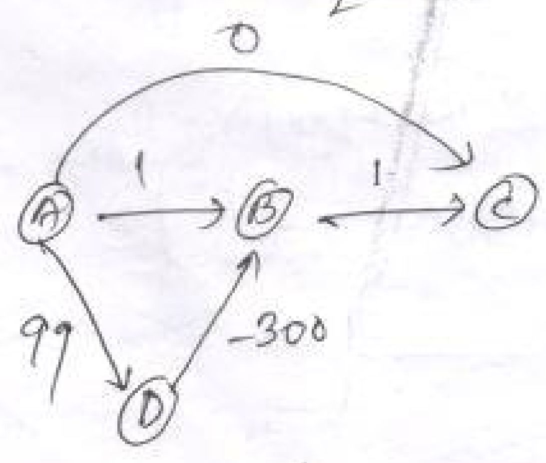
→ Incase of -ve cycle Bellman-ford algorithm ~~can~~ fails but it can detect the if there is -ve cycle in the graph.

→ Dijkstra algorithm is Greedy approach while Bellman-ford is Dynamic approach.

→ worst case time complexity of Dijkstra is $O(n^2)$ (v log v) where as for Bellman ford it is $O(n^3)$

Q why Dijkstra can not handle -ve weight?



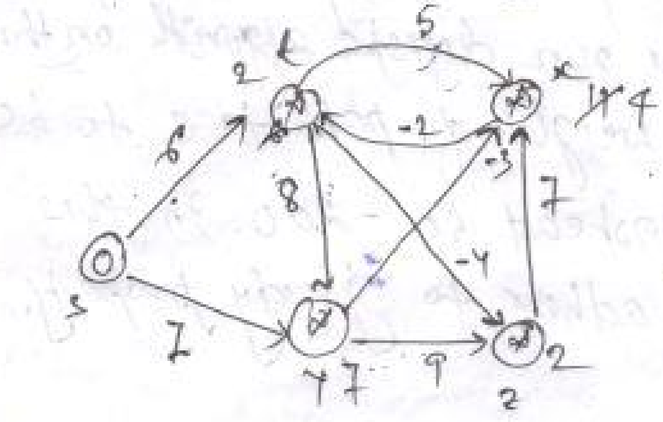


⇒ Here we want to find shortest path tree from source node A. If you run dijistra algorith on this, we get the minimum weight of path to C to be which should instead be -200. In this case graph does not adhere to greedy property.

eas

BELLMAN-FORD(G, W, S)

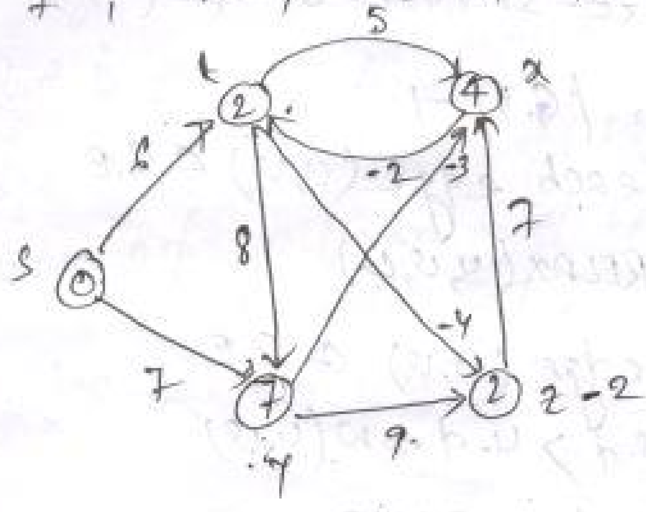
1. INITIALIZE-SINGLE-SOURCE(G, S)
2. for i=1 to |G.V|-1
3. for each edge (u,v) ∈ G.E
4. RELAX(u,v,w)
5. for each edge (u,v) ∈ G.E
6. if v.d > u.d + w(u,v)
7. return FALSE.
8. ~~RETUR~~ return TRUE.



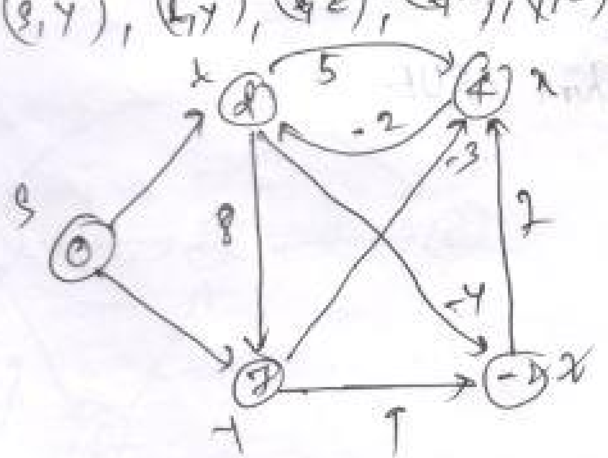
①

(s,t), (s,y), (t,y), (t,z), (t,x), (y,z), (y,x), (x,t

6, 7, 7, 2, 11, 2, 4, 2



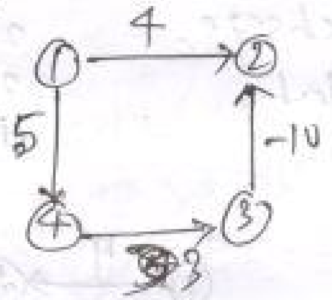
② (s,t), (s,y), (t,y), (t,z), (t,x), (y,z), (y,x), (x,



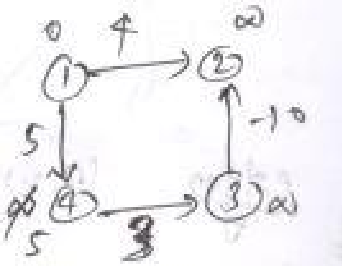
③ (s,t), (s,y), (t,y), (t,z), (t,x) (y,z), (y,x), (z,t), (z,x)

④
Since the state is not changing, so we can stop here, But algorithm will run for |v|-1 times.

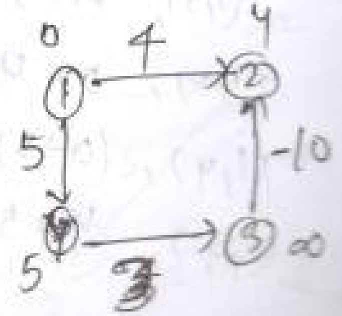
Practice



edges, ~~(1,2), (1,4),~~
(3,2), (4,3), (1,4), (1,2)



1st for (3,2), $\infty - 10 < \infty = \infty$
(4,3), $\infty + 3 < \infty = \infty$
(1,4), $0 + 5 < \infty = 5$
(1,2), $0 + 4 < \infty = 4$



2nd for (3,2), $\infty - 10 = \infty = \infty$
(4,3), $5 + 3 < \infty = 8$
(1,4), $0 + 5 < 5 = 5$
(1,2), $0 + 4 < 4 = 4$

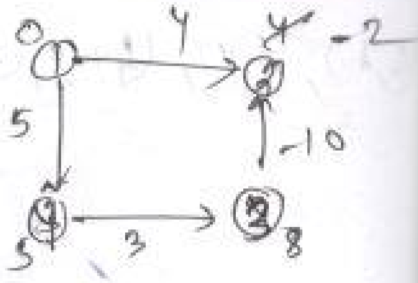
3rd

for (3,2), $8-10 < 4 = -2$

for (4,3), $5+3 < 8 = 8$

for (1,4), $0+5 < 5 = 5$

for (1,2), $0+4 < -2 = -2$

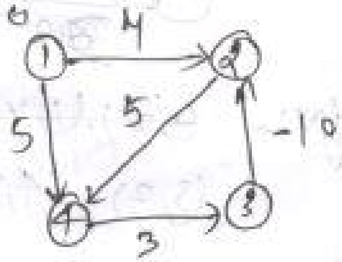


Relex one more time,
there will be no change,
so answer is perfect

Shortpath

1 - 0
2 - (-2)
3 - 8
4 - 5

Ex 2



edges (3,2), (4,3), (1,4), (1,2) (2,4)

1st

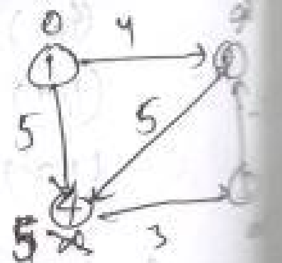
for (3,2), $\infty - 10 < \infty = \infty$

(4,3), $\infty + 3 < \infty = \infty$

(1,2), $0 + 4 < \infty = 4$

(1,4), $(0+5) < \infty = 5$

(2,4), $4+5 < 4 = 4$



2nd

for

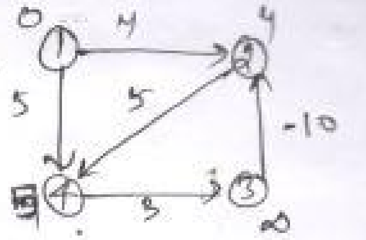
(3,2) $\infty - 10 < 4 = 4$

(4,3) $5+3 < 8 = 8$

(1,2) $0+4 < 4 = 4$

(1,4), $0+5 < 5 = 5$

(2,4), $4+5 < 5 = 5$



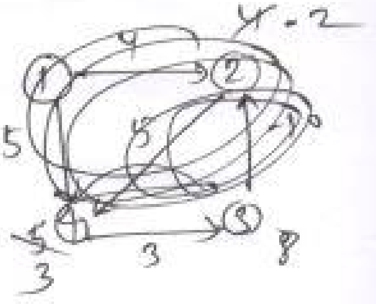
3rd

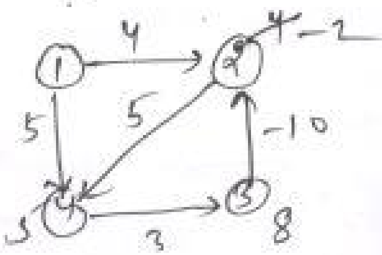
~~(3,2) 8-10 < 4 = -2~~

~~(4,3) -2+5 < 5 = 3~~

~~(1,2) 0+4 < -2 = -2~~

~~(1,4)~~





for (3,2) $8-10 < 4 = -2$

(4,3) $5+3 < 8 = 8$

(1,2) $0+4 < -2 = -2$

(1,4) $0+5 < 5 = 5$

(2,4) $-2+5 < 5 = 3$

Done, do one more time.

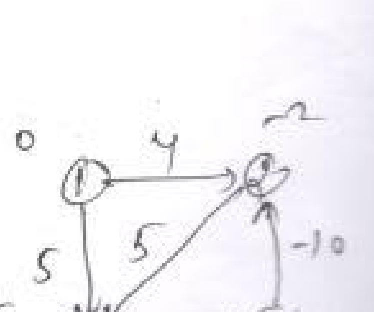
for (3,2), $8-10 < -2 = -2$

(4,3), $3+3 < 8 = 6$ } changing again

(1,2) $0+4 < -2 = -2$

(1,4) $0+5 < 3 = 3$

(2,4) $-2+5 < 3 = 3$



Hence Bellman ford algorithm cannot handle -ve cycle