Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Slope and Deflection (Superposition, Maxwell Reciprocal and Betti's Theorem)
Lecture: 09
Course Instructor: Prof. Rashid Mustafa

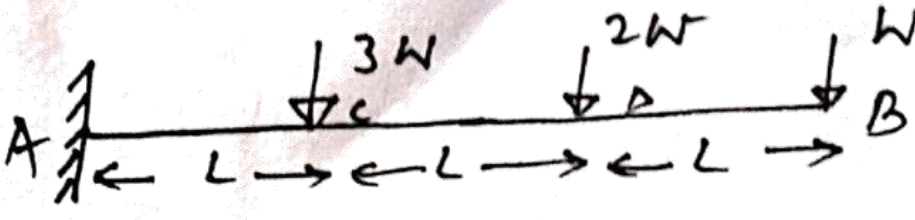
Superposition Method :

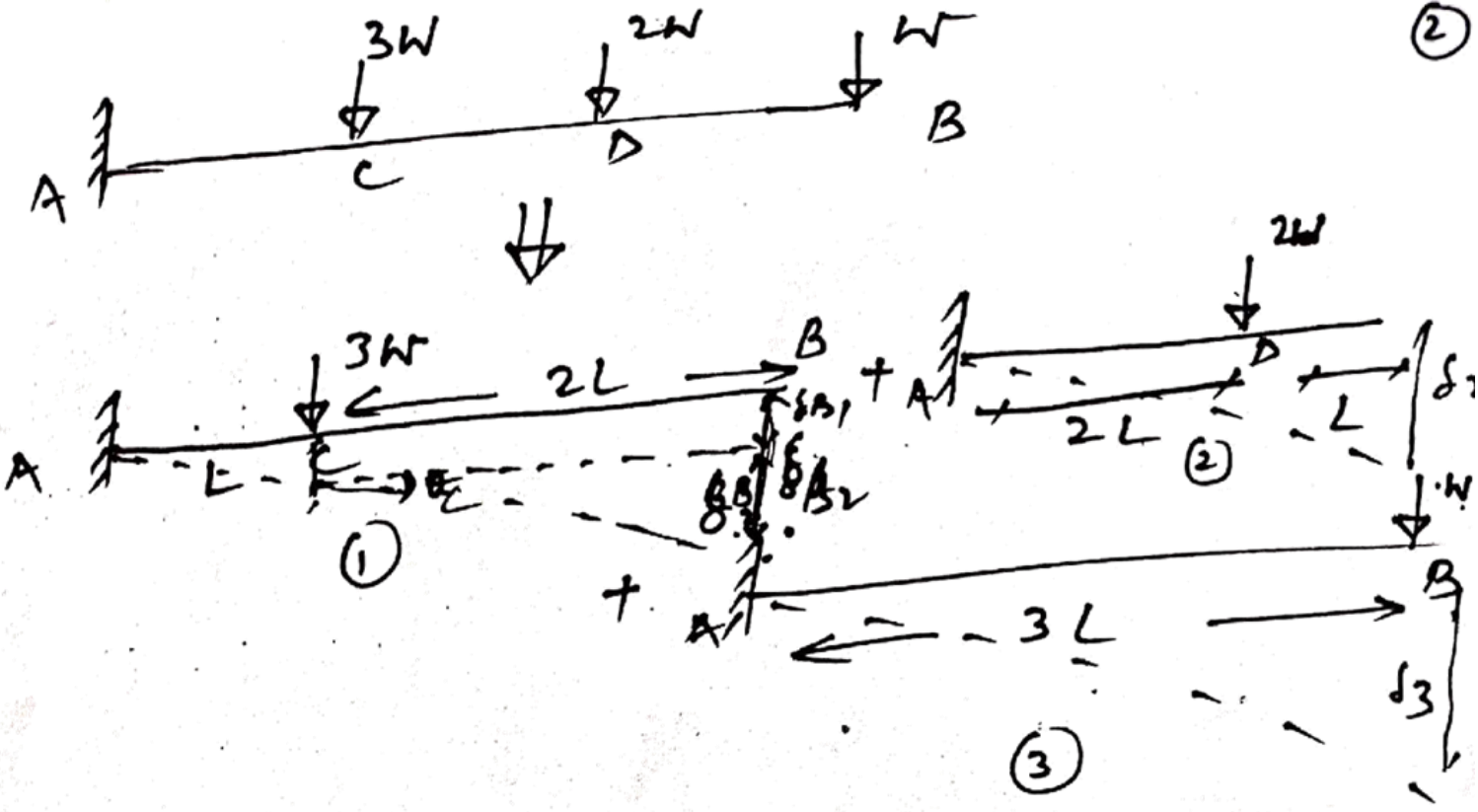
→ Method is suitable when More than one types of loads are acting and for each load deflection and slopes at the Concerne point is known by standard result.

→ Final deflections & slopes is obtained by superpositions of all the results.

→ It is not a Basic method.

Q-1. A Cantilever Beams in which Concentrated point load is acting as shown in figure. Find out the slopes and Deflections at the free end.





$$\delta_B = \delta_1 + \delta_2 + \delta_3$$

In beam (1)

$$\delta_{B_1} = \delta_C = \frac{(3W)L^3}{3EI} = \frac{WL^3}{EI}$$

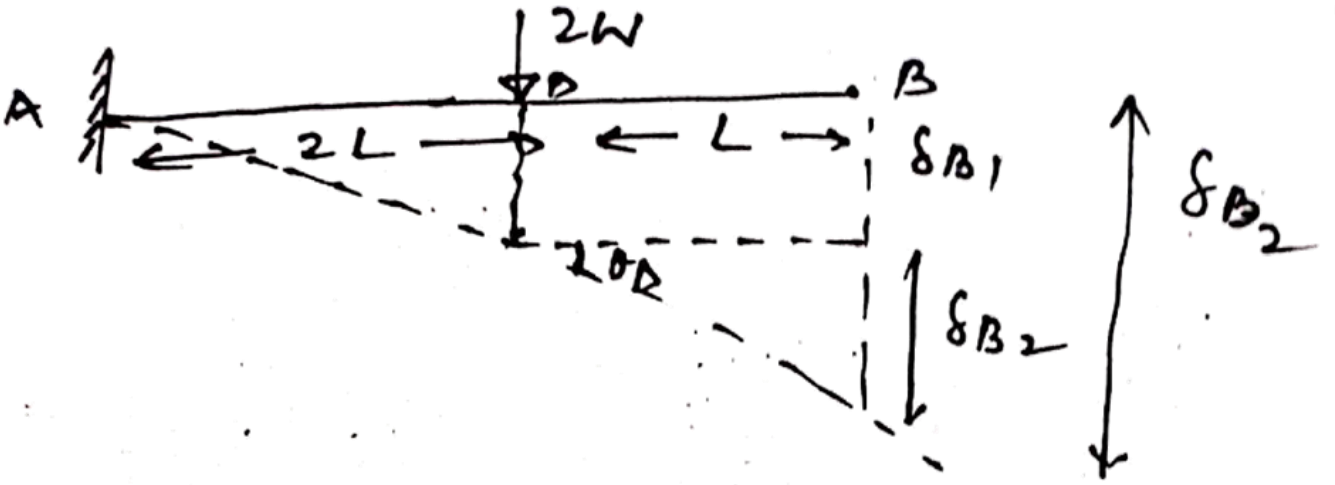
$$\theta_C = \frac{WL^2}{2EI} = \frac{(3W)\cdot L^2}{2EI} = \frac{3WL^2}{2EI}$$

$$\delta_{B_2} = \theta_C \times 2L$$

$$= \left(\frac{3WL^2}{2EI}\right) \times 2L = \frac{3WL^3}{2EI}$$

$$(\delta_B)_1 = \delta_{B_1} + \delta_{B_2}$$

$$= \frac{3WL^3}{EI} + \frac{3WL^3}{2EI} = \frac{9WL^3}{2EI}$$



$$\delta_D = \frac{WL^3}{3EI} = \frac{(2W) \times (2L)^3}{3EI}$$

$$= \frac{16WL^3}{3EI}$$

$$\theta_D = \frac{WL^2}{2EI} = \frac{(2W) \times (2L)^2}{2EI}$$

$$= \frac{4WL^2}{EI}$$

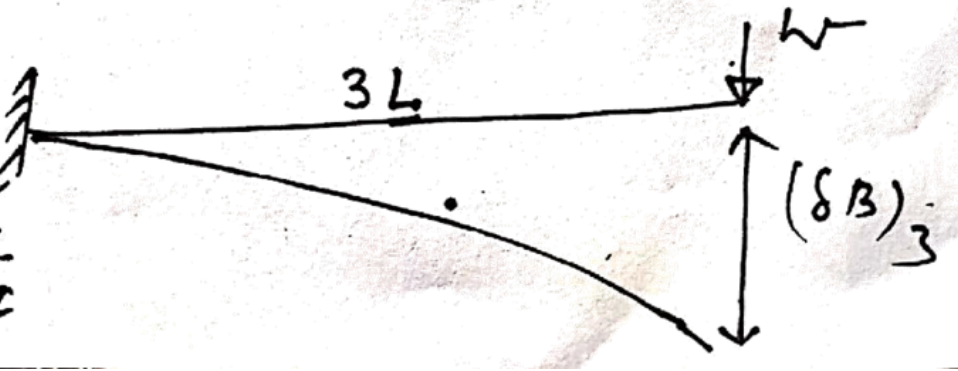
$$(\delta_B)_2 = \delta_{B_1} + \delta_{B_2}$$

$$= \frac{16WL^3}{3EI} + \theta_D \times L$$

$$= \frac{16WL^3}{3EI} + \frac{4WL^2}{EI} \times L$$

$$= \frac{28WL^3}{3EI}$$

In beam (3)



$$(\delta_B)_3 = \frac{WL^3}{3EI}$$

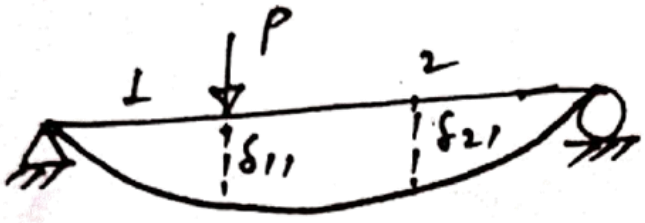
$$(\delta B)_3 = \frac{W(3L)^3}{3EI} = \frac{9WL^3}{EI}$$

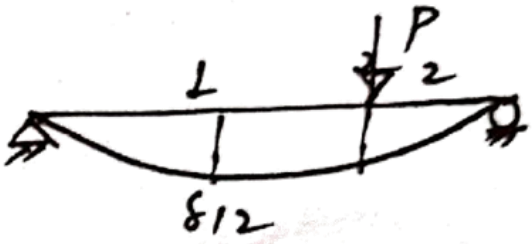
$$\text{Total deflection at B} = (\delta B)_1 + (\delta B)_2 + (\delta B)_3$$

$$= \frac{4WL^3}{EI} + \frac{24WL^3}{3EI} + \frac{9WL^3}{EI}$$

$\Rightarrow$ MAXWELL RECIPROCAL THEOREM

Case 1.





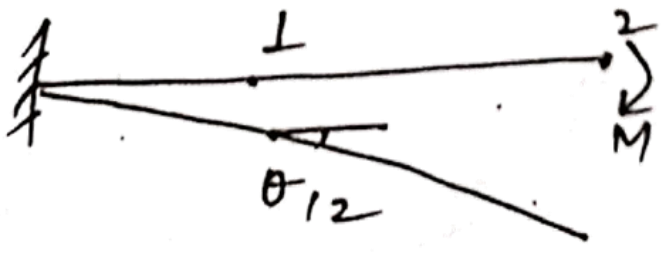
δ_{11} → Deflection at 1 due to load at P

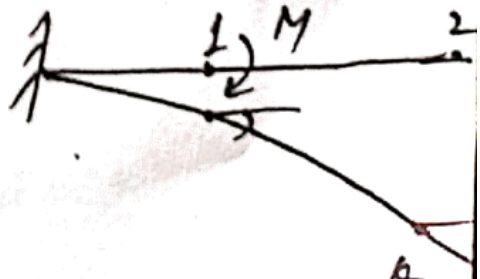
δ_{21} → Deflection at 2 due to load at 1

δ_{12} = Deflection at 1 due to load at 2

$$\boxed{\delta_{12} = \delta_{21}}$$

Case 2.



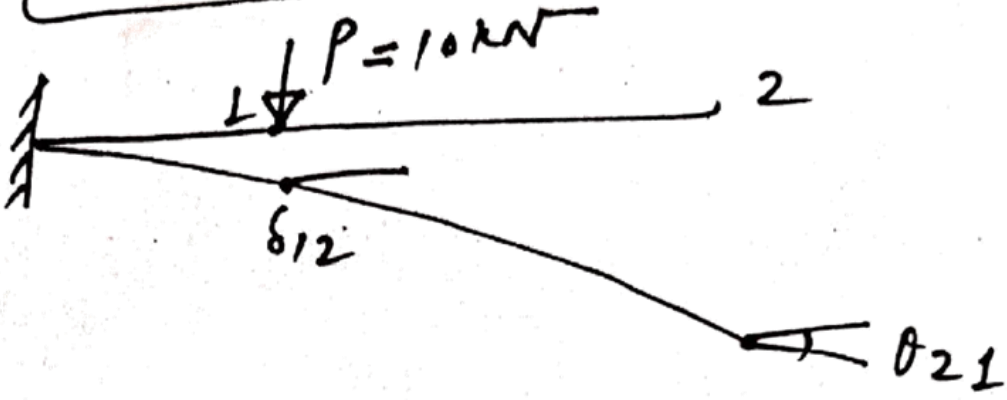


θ_{12} → Slope at 1 due to Moment at 2

θ_{21} → Slope at 2 due to Moment at 1.

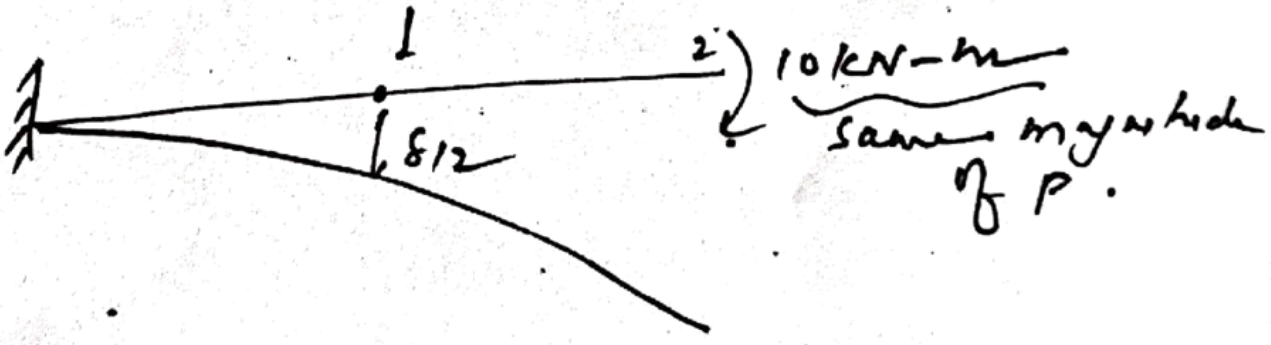
$$\boxed{\theta_{12} = \theta_{21}}$$

Case 3.



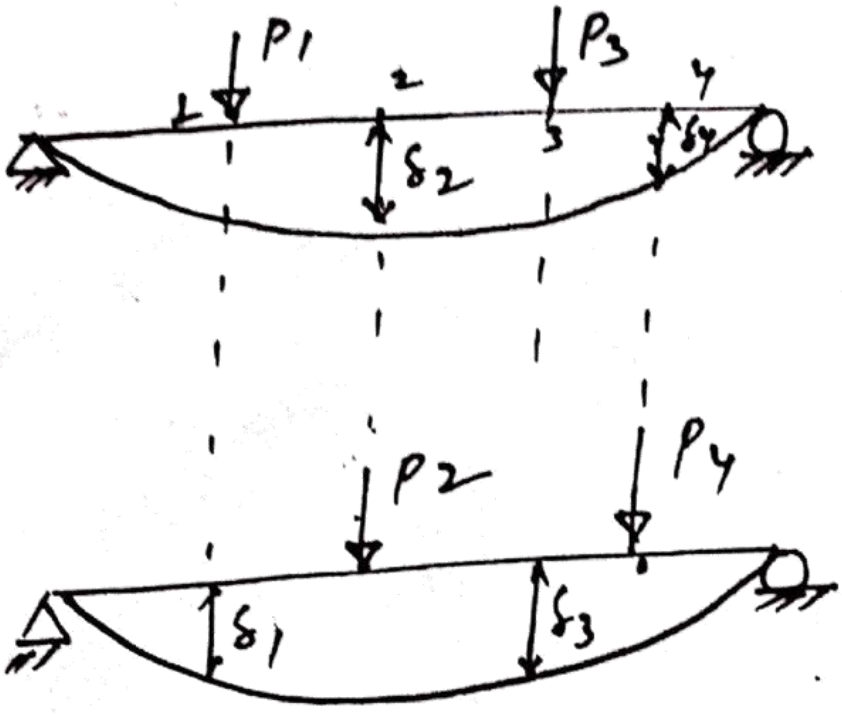
δ_{12} = Deflection at 1 due to moment at 2.

θ_{21} → Slope at 2 due to load at 1



$$\boxed{\theta_{21} = \delta_{12}}$$

⇒ Betti's Law



$$P_1\delta_1 + P_3\delta_3 = P_2\delta_2 + P_4\delta_4$$

↳ A/c to Betti's Law

$\delta_1, \delta_3 \rightarrow$ deflections at point (1) & (3) due to load P_2 & P_4

$\delta_2, \delta_4 \rightarrow$ Deflections at point (2) & (4) due to loads P_1 and P_3

HAPPY AND SAFE LEARNING