

# Travelling salesman problem (Held-Karp Dp algorithm)

1.  $C(\{1\}, 1) = 0$

2. for  $s=2$  to  $n$

3. for all subsets  $S \subseteq \{1, 2, \dots, n\}$  of size  $s$  and

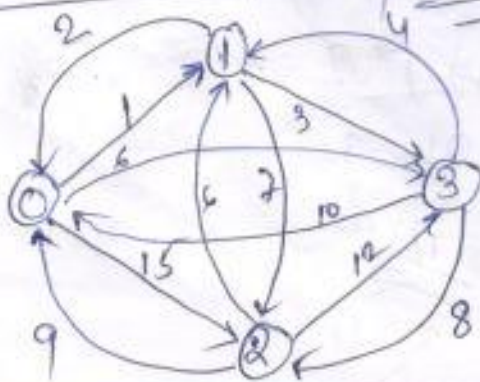
4.  $C(S, 1) = \infty$

5. for all  $j \in S, j \neq 1$ :

6.  $C(S, j) = \min_{i \in S, i \neq j} \{ C(S - \{j\}, i) + d_{ij} \}$

7. return  $\min_j C(\{1, \dots, n\}, j) + d_{j1}$

Time complexity  $O(n^2 \cdot 2^n)$



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	0	1	2	3
0	0	1	15	6
1	2	0	7	3
2	9	6	0	12
3	10	4	8	0

→ Generate all subset except starting vertex.

- $\{\emptyset\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}$
- $\{2,3\}, \{1,2,3\}$

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Now, we want to traverse all the vertex, except the starting vertex and vertexes in the subset using these subset.

→ let's proceed, traverse all the vertex using subset  $\{\emptyset\}$  except ~~vertex~~ starting vertex.

$\{1, \{\emptyset\}\} = \text{cost}[0,1] + \text{cost}[0,0] = 1 + 0 = 1^{(1)}$

$\{2, \{\emptyset\}\} = \text{cost}[0,2] + \text{cost}[0,0] = 15 + 0 = 15^{(1)}$

$\{3, \{\emptyset\}\} = \text{cost}[0,3] + \text{cost}[0,0] = 6 + 0 = 6^{(1)}$

→ Now traverse all the vertex using subset  $\{1\}$  except the starting vertex i.e 0 and vertex in subset i.e 1.

$\{2, \{1\}\} = \text{cost}[1,2] + \text{cost}[1, \{\emptyset\}] = 7 + 1 = 8^{(1)}$

$\{3, \{1\}\} = \text{cost}[1,3] + \text{cost}[1, \{\emptyset\}] = 3 + 1 = 4^{(1)}$

→ let's we reach vertex 2 from vertex 1. Now we have reach 1 from starting vertex i.e 0 via remaining subset i.e  $\{\emptyset\}$  so technically it is the sum of  $\text{cost}[1,2] + \text{cost}[1, \{\emptyset\}]$

cost of reaching each remaining subset

Similarity

$$\{1, \{2, 2\}\} = \text{cost}[2, 1] + \text{cost}[2, \{2, \emptyset\}] = 6 + 15 = 21 \quad (2)$$

$$\{3, \{2, 2\}\} = \text{cost}[2, 3] + \text{cost}[2, \{2, \emptyset\}] = 12 + 15 = 27 \quad (2)$$

$$\{1, \{3, 3\}\} = \text{cost}[3, 1] + \text{cost}[3, \{3, \emptyset\}] = 4 + 6 = 10 \quad (3)$$

$$\{2, \{3, 3\}\} = \text{cost}[3, 2] + \text{cost}[3, \{3, \emptyset\}] = 8 + 6 = 14 \quad (3)$$

$$\{0, \{2, 3, 3\}\} = \min \begin{cases} \text{cost}[2, 1] + \text{cost}[2, \{3, 3\}] = 6 + 14 \\ \text{cost}[3, 1] + \text{cost}[3, \{2, 3\}] = 4 + 23 \end{cases}$$

$\Rightarrow 0 + 1 = 1$   
 $\Rightarrow 0 + 21 = 21$

$$= 20 \quad (2)$$

$$\{2, \{1, 3, 3\}\} = \min \begin{cases} \text{cost}[1, 2] + \text{cost}[1, \{3, 3\}] = 7 + 10 = 17 \\ \text{cost}[3, 2] + \text{cost}[3, \{1, 3\}] = 8 + 4 = 12 \end{cases}$$

1st index = 12 (3)

$$\{3, \{1, 2, 2\}\} = \min \begin{cases} \text{cost}[1, 3] + \text{cost}[1, \{2, 2\}] = 7 + 21 = 28 \\ \text{cost}[2, 3] + \text{cost}[2, \{1, 2\}] = 12 + 8 = 20 \end{cases}$$

$\Rightarrow 1 + 0 = 1$   
 $\Rightarrow 20 = 20 \quad (2)$

$$\{0, \{1, 2, 3, 3\}\} = \min \begin{cases} \text{cost}[1, 0] + \text{cost}[1, \{2, 3, 3\}] = 2 + 20 = 22 \\ \text{cost}[2, 0] + \text{cost}[2, \{1, 2, 3\}] = 9 + 12 = 21 \\ \text{cost}[3, 0] + \text{cost}[3, \{1, 2, 2\}] = 10 + 20 = 30 \end{cases}$$

$$= 21 \quad (2)$$

so we reach vertex 0 from 2. Now we have to  
 find how did we reach 2 via subset  $\{1, 3\}$ .  
 i.e.  $\{2, \{1, 3\}\}$ , so in the solution we see that  
 we reach 2 from vertex 3 ( $0 \leftarrow 2 \leftarrow 3 + \{1\}$ ). so now  
 we have to find how did we reach 3 via subset  $\{1, 3\}$   
 so we found that we reach 3 from 1. so the  
 order of traversal is

$0 \leftarrow 2 \leftarrow 3 \leftarrow 1 \leftarrow 0$



## The Bellman-Ford algorithm

→ Dijkstra algorithm works only on non-negative weight where as Bellman-ford can work with -ve weight.

→ In case of -ve cycle Bellman-ford algorithm ~~can~~ fails but it can detect the if there is -ve cycle in the graph.

→ Dijkstra algorithm is greedy approach where Bellman-ford is dynamic approach.

⇒ worst case time complexity of Dijkstra is  $(V \log V)$  where as for Bellman-ford it is  $O(V^3)$

⊛ why Dijkstra can not handle -ve weight?

