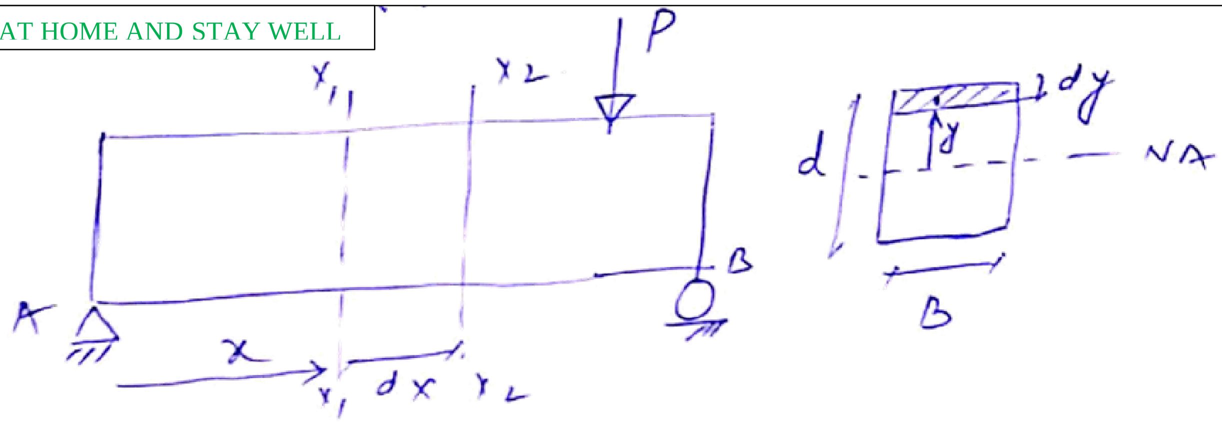
Department of Civil Engineering
Katihar Engineering College, Katihar

Subject : Introduction to Solid Mechanics
Topic : Shear Stress in Beam
Lecture : 01
Course Instructor : Prof. Rashid Mustafa

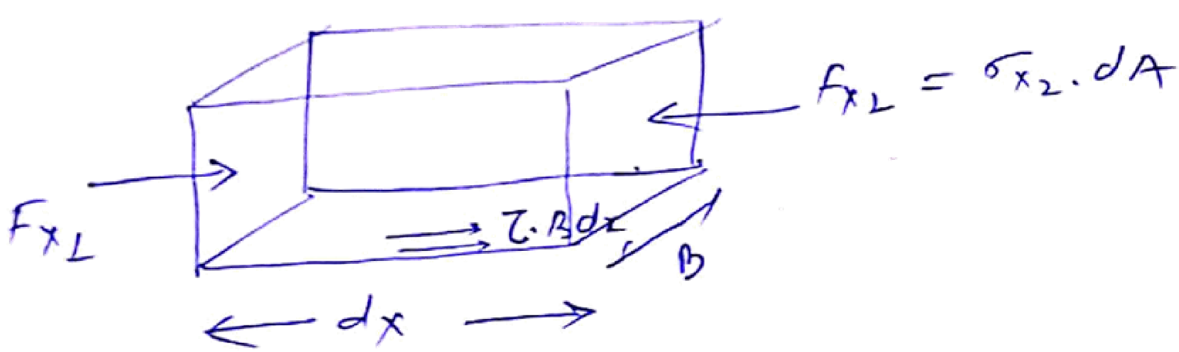


$M_{x_1} = V_A \cdot x$ $\quad (M_{x_2} > M_{x_1})$

$M_{x_2} = V_A (x + dx)$

Ac to Bending Eqn

$$\sigma = \frac{M \cdot y}{I} \Rightarrow \sigma_{x_1} = \frac{M_{x_1} \cdot y}{I}$$



$$\sigma_{x_2} = \frac{(M + dM)}{I} \cdot y$$

$$F_{x1} + \tau \cdot B \cdot dx = F_{x2}$$

$$\tau \cdot B \cdot dx = \left[\left(\frac{M + dM}{I}\right) \cdot y - \frac{M \cdot y}{I}\right] dA$$

$$\tau \cdot B \cdot dx = \frac{dM}{I} \cdot y \cdot dA$$

$$\tau = \left(\frac{dM}{dx}\right) \cdot \frac{y \cdot dA}{I \cdot B}$$

$$\tau = \frac{V}{IB} \cdot y \, dA$$

$$\boxed{\tau = \frac{V}{I \cdot B} \cdot A \cdot \bar{y}}$$

Where
$V \rightarrow$ Shear force at the Section where shear stress measured

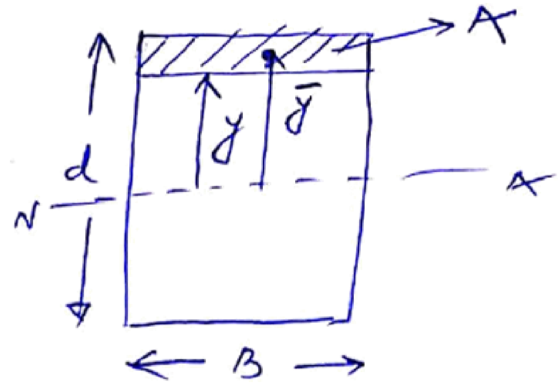
$I \rightarrow$ Moment of inertia of x-section about N.A

$B \rightarrow$ Width of x-section at that point where shear stress measured

$A \rightarrow$ Area of the point (Above or below) where shear stress measured.

$\bar{y} \rightarrow$ Distance of Centroid of above area from the Neutral axis

⇒ Shear stress in Rectangular section



$$I = \frac{Bd^3}{12}$$

$$\tau = \frac{V}{IB} \cdot A \cdot \bar{y}$$

$$A \cdot \bar{y} = B\left(\frac{d}{2} - y\right) \times \left[y + \frac{\frac{d}{2} - y}{2}\right]$$

$$A\bar{y} = B\left(\frac{d}{2} - y\right)\left(y + \frac{d}{2}\right)/2$$

$$A\bar{y} = \frac{B}{2}\left(\frac{d^2}{4} - y^2\right)$$

$$\text{Shear stress}(\tau) = \frac{V}{I \cdot B} \cdot A\bar{y}$$

$$= \frac{V \cdot B\left(\frac{d^2}{4} - y^2\right)}{2 \times \frac{Bd^3}{12} \times B}$$

$$\boxed{\text{Shear stress}(\tau) = \frac{6V}{Bd^3}\left(\frac{d^2}{4} - y^2\right)}$$

$$\boxed{\text{Shear stress}(\tau) = \frac{V}{2I}\left(\frac{d^2}{4} - y^2\right)}$$

→ Maximum shear stress occurs at $y = 0$ (i.e at Neutral axis)

$$\rightarrow \tau_{max} = \frac{6V}{Bd^3}\left(\frac{d^2}{4}\right) = \frac{3}{2}\frac{V}{Bd}$$

Scanned with CamScanner

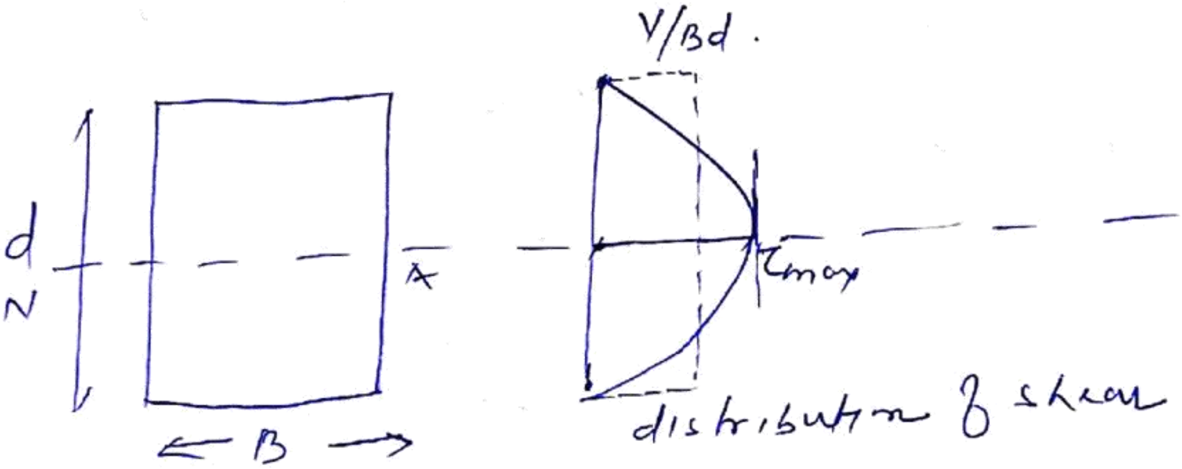
$$\frac{V}{Bd} = \tau_{avg}$$

$$\boxed{\tau_{avg} = \frac{V}{Bd}}$$

→ $\tau_{max} = \frac{3}{2}\frac{V}{Bd} = \frac{3}{2}\tau_{avg}$

→ $\boxed{\tau_{max} = 1.5\,\tau_{avg}}$

→ The variation of shear stress is parabolic.



distribution of shear stress.

$$\tau = \frac{6V}{Bd^3}\left(\frac{d^2}{4} - y^2\right)$$

When $y = d/2$, $\tau = 0$

When $y = 0$, $\tau = \tau_{NA} = \tau_{max} = 1.5\,\tau_{avg}$

→ Maximum shear stress in a rectangular beam occurs at Neutral axis and magnitude of maximum shear stress is 1.5 times the average shear stress.

Normal shear stress = Average shear stress.

$$\frac{6V}{Bd^3}\left(\frac{d^2}{4}-y^2\right) = \frac{V}{Bd}$$

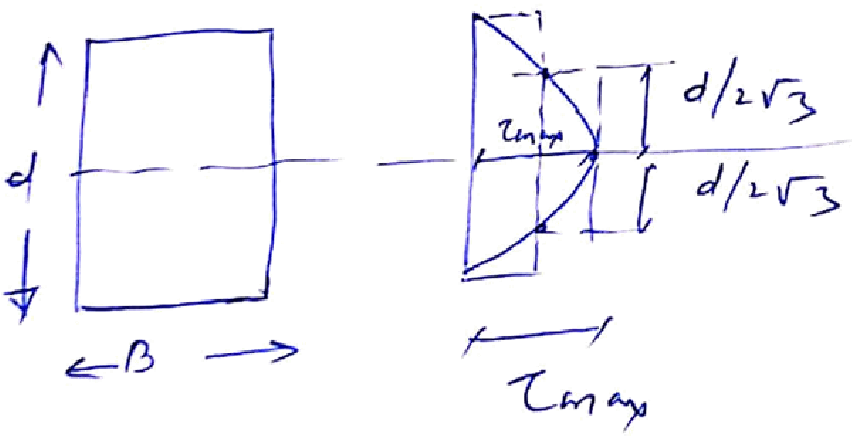
$$\frac{6}{d^2}\left(\frac{d^2}{4}-y^2\right) = 1$$

$$\frac{d^2}{4}-y^2 = \frac{d^2}{6}$$

$$y^2 = \frac{d^2}{12}$$

$$\boxed{y = \frac{d}{2\sqrt{3}}}$$

At $y = \frac{d}{2\sqrt{3}}$ from the Neutral axis average shear stress is equal to Normal shear stress.



Distribution of shear stress (Parabolic)

HAPPY LEARNING