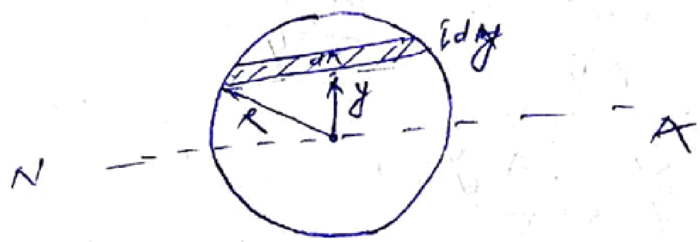
Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Introduction to Solid Mechanics
Topic: Shear Stress in Beam
Lecture: 02
Course Instructor: Prof. Rashid Mustafa

(2) Circular Section:



Shear stress at a distance y from N.A $= \frac{V}{IB} \cdot A\bar{y}$

$$A \cdot \bar{y} = \int_y^R y \cdot dA = \int_y^R y \times (2\sqrt{R^2 - y^2}) \cdot dy$$

$$A \cdot \bar{y} = \int_y^R 2y\sqrt{R^2 - y^2}\, dy$$

Let $R^2 - y^2 = x^2$

$$-2y\, dy = 2x\, dx$$

When $y = y$, $x = \sqrt{R^2 - y^2}$

When $y = R$, $x = 0$

$$A\bar{y} = \int_{\sqrt{R^2-y^2}}^{0} \frac{-1}{\cancel{\sqrt{R^2-y^2}}}\, 2x\,dx\,(x)$$

$$A\bar{y} = -\frac{2}{3}\, x^3 \Big|_{\sqrt{R^2-y^2}}^{0}$$

$$= -\frac{2}{3}\left[0-(R^2-y^2)^{3/2}\right]$$

$$= \frac{2}{3}(R^2-y^2)^{3/2}$$

$$A\bar{y} = \frac{2}{3}(R^2-y^2)^{3/2}$$

$$I = \frac{\pi D^4}{64} = \frac{\pi R^4}{4}$$

$$\tau = \frac{V}{IB}\cdot A\bar{y}$$

$$= \frac{V\times\frac{2}{3}(R^2-y^2)^{3/2}}{\left(\frac{\pi R^4}{4}\right)\times 2\sqrt{R^2-y^2}}$$

Shear stress $= \frac{4V}{3}\left(\frac{R^2-y^2}{R^2}\right)\cdot\frac{1}{\pi R^2}$

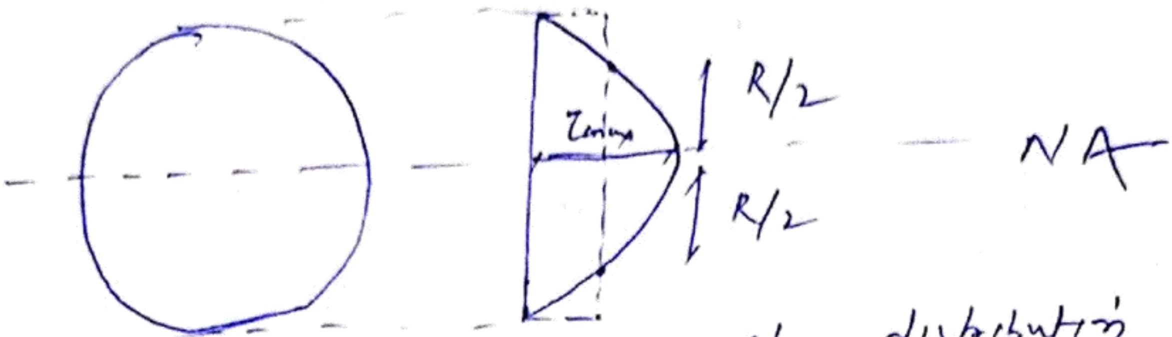
$$\boxed{\tau = \frac{4}{3}\left(\frac{V}{\pi R^2}\right)\left(1-\frac{y^2}{R^2}\right)}$$

$\boxed{\text{Average shear stress } (\tau_{avg}) = \frac{V}{\pi R^2}}$

$$\boxed{\tau = \frac{4}{3}\,\tau_{avg}\left(1-\frac{y^2}{R^2}\right)}$$

$$\boxed{\text{Maxm shear stress } (\tau_{max}) = \frac{4}{3}\tau_{avg}}$$

→ Maximum shear stress occurs at the N.A (i.e $y=0$)



- Shear stress distribution

Normal shear stress = Avg shear stress

$$\frac{4}{3}\cancel{\tau_{avg}}\left(1-\frac{y^2}{R^2}\right) = \frac{V}{\cancel{\pi} R^2}$$

$$4 - \frac{4y^2}{R^2} = 3$$

$$\frac{4y^2}{R^2} = 1$$

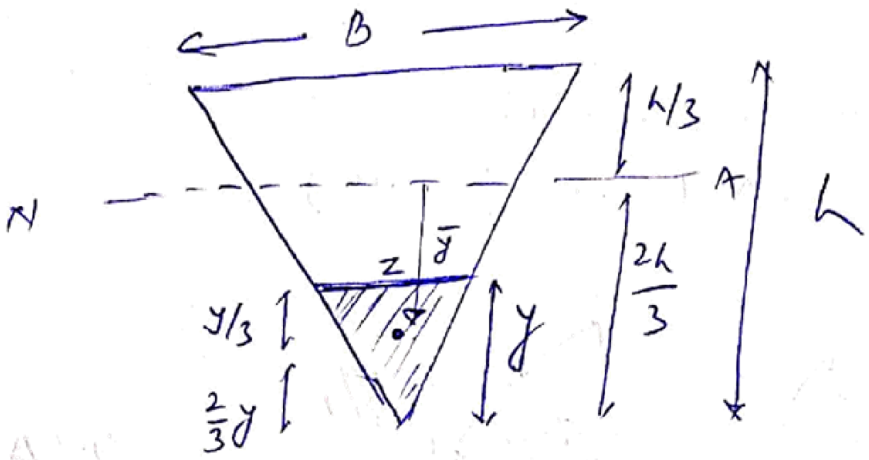
$$\boxed{y = R/2}$$

→ Maximum shear stress occurs in a Circular section is at Neutral axis (i.e $y=0$)

$$\boxed{\tau_{max} = \frac{4}{3}\tau_{avg}}$$

→ Normal shear stress = Average shear stress at $y = R/2$ from N.A

(3) Triangular Section:



$$\text{Shear stress}(\tau) = \frac{V}{I \cdot B} \cdot A\bar{y}$$

$$= \frac{V}{I z} A \cdot \bar{y}$$

$$\text{Area}(A) = \frac{1}{2} \times y \times z = \frac{yz}{2}$$

$$\bar{y} = \frac{2}{3}h - \frac{2}{3}y$$

$$= \frac{2}{3}(h - y)$$

$$\text{Shear stress}(\tau) = \frac{V}{I z} \cdot A \cdot \bar{y}$$

$$= \frac{V}{I \cdot z} \frac{yz}{2} \times \frac{2}{3}(h-y)$$

$$\boxed{\tau = \frac{V \cdot y}{3I}(h-y)}$$

↳ Parabolic

For Maximum Shear Stress

$$\frac{d\tau}{dy} = 0$$

$$h - 2y = 0$$

$$\boxed{y = \frac{h}{2}}$$

→ ∴ Maxm shear stress occur at mid height of triangular section (i.e $y = h/2$)

Variation of shear stress

$$\tau = \frac{V \cdot y}{3I}(h - y)$$

At NA ($y = \frac{2}{3}h$)

$$\tau = \frac{2V \cdot h^2}{\cancel{27} \times \frac{Bh^3}{36}} = \frac{72}{27} \cdot \frac{V}{Bh}$$

$$\tau = \frac{72}{27} \cdot \frac{V}{Bh}$$

$$= \frac{8}{3} \frac{V}{Bh} = \frac{4}{3} \cdot \frac{V}{\left(\frac{Bh}{2}\right)}$$

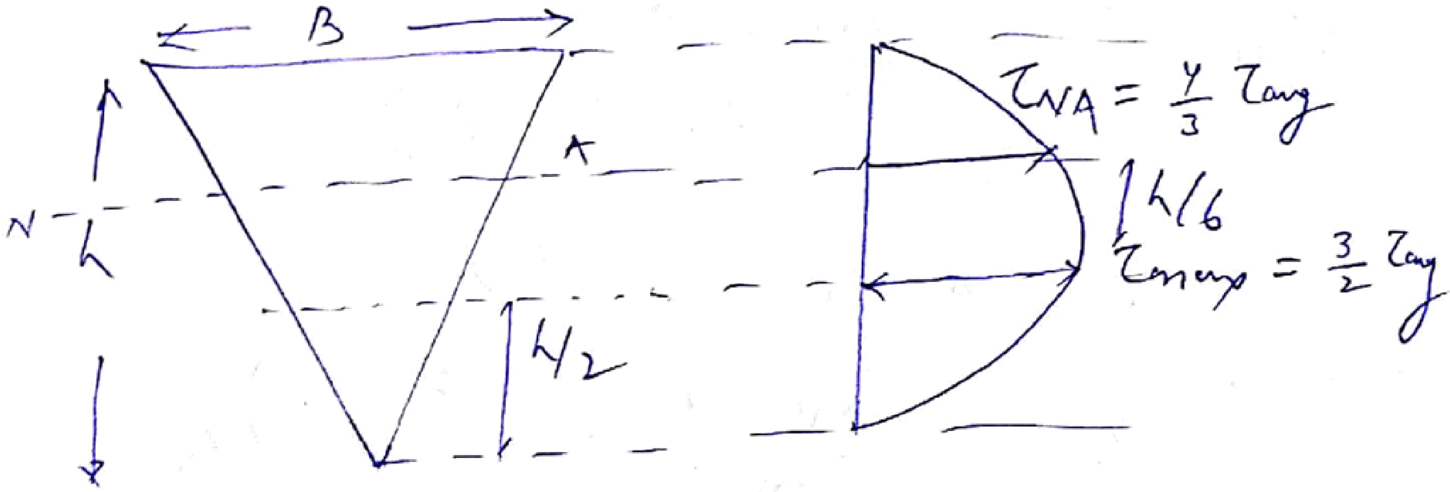
$$\boxed{\tau_{NA} = \frac{4}{3} \tau_{avg}}$$

Where $\boxed{\text{Average shear stress } (\tau_{avg}) = \frac{V}{A} = \frac{V}{\frac{Bh}{2}}}$

$$\tau_{max} = \frac{V \cdot h}{2 \times 3 \times \frac{Bh^3}{36}} \left(h - \frac{h}{2} \right)$$

$$= 3 \frac{V}{Bh} = \frac{3}{2} \left(\frac{V}{\frac{Bh}{2}} \right)$$

$$\boxed{\tau_{max} = 1.5\, \tau_{avg}}$$



Distribution of Shear stress.

Key Points:

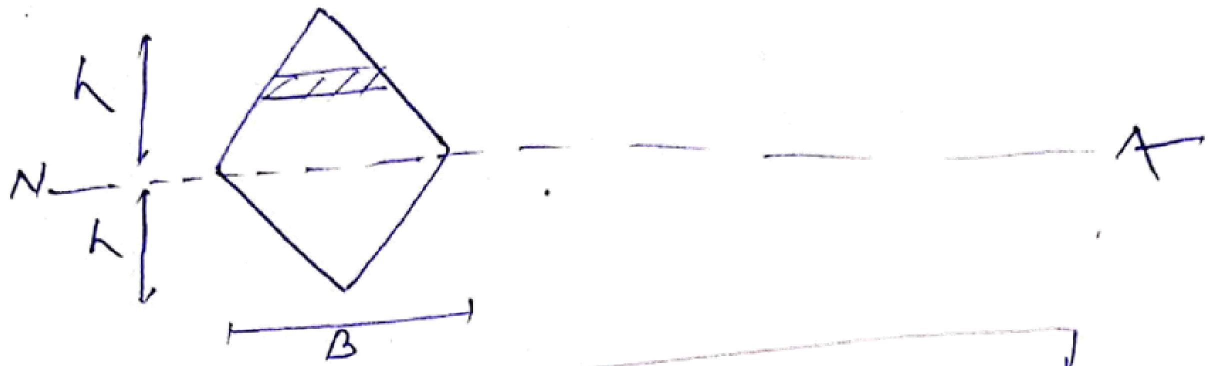
(i) Maximum shear stress occurs at $y = h/2$.

(ii) $\tau_{max} = 1.5\, \tau_{avg}$.

(iii) $\tau_{NA} = \frac{4}{3} \tau_{avg}$.

(iv) Distance b/w N.A & τ_{max} = $h/6$

(4) Diamond Section or Square Section in which one diagonal is horizontal.



$$\boxed{\tau = \frac{V}{Bh^3}(h-y)(2y+h)}$$

$$\boxed{\tau_{NA} = \frac{V}{Bh}.}$$

$$\boxed{\text{Maximum shear stress } (\tau_{max}) = \frac{9}{8}\tau_{avg}}$$

→ Maximum shear stress occurs at a distance of $y = h/4$

Shear stress distribution