Department of Civil Engineering
Katihar Engineering College, Katihar

Subject: Soil & Rock Mechanics
Topic: Shear Strength of Soil
Lecture: 01
Course Instructor: Prof. Rashid Mustafa

⇒ Shear strength of a soil is the resistance offered by the soil grains against shear deformation.

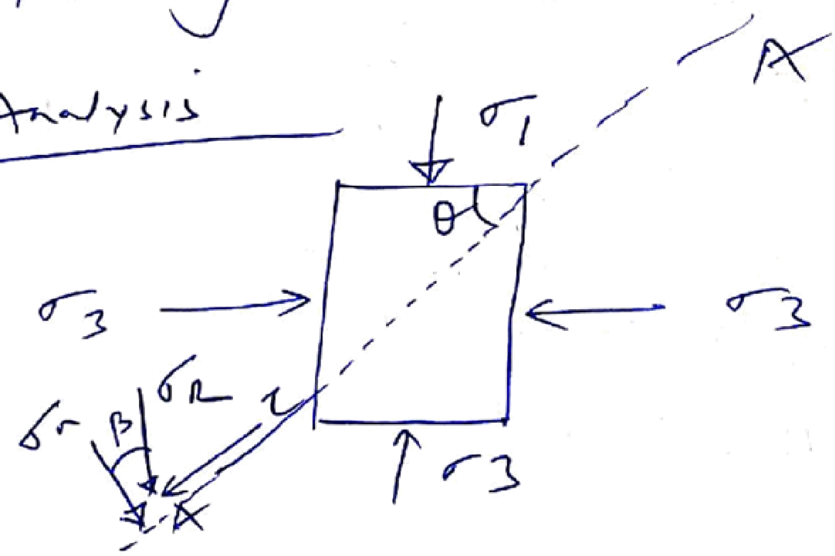
⇒ Example of shear failure

(a) failure of infinite slope

(b) failure of finite slope

(c) failure of soil below a building foundation (General shear failure, Local shear failure & punching shear failure)

2-D Analysis



Consider a plane A–A at angle θ with major Principal plane

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta$$

$$\tau = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2 + 2 \times \sigma_n \times \tau \times \cos\theta}$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau^2}$$

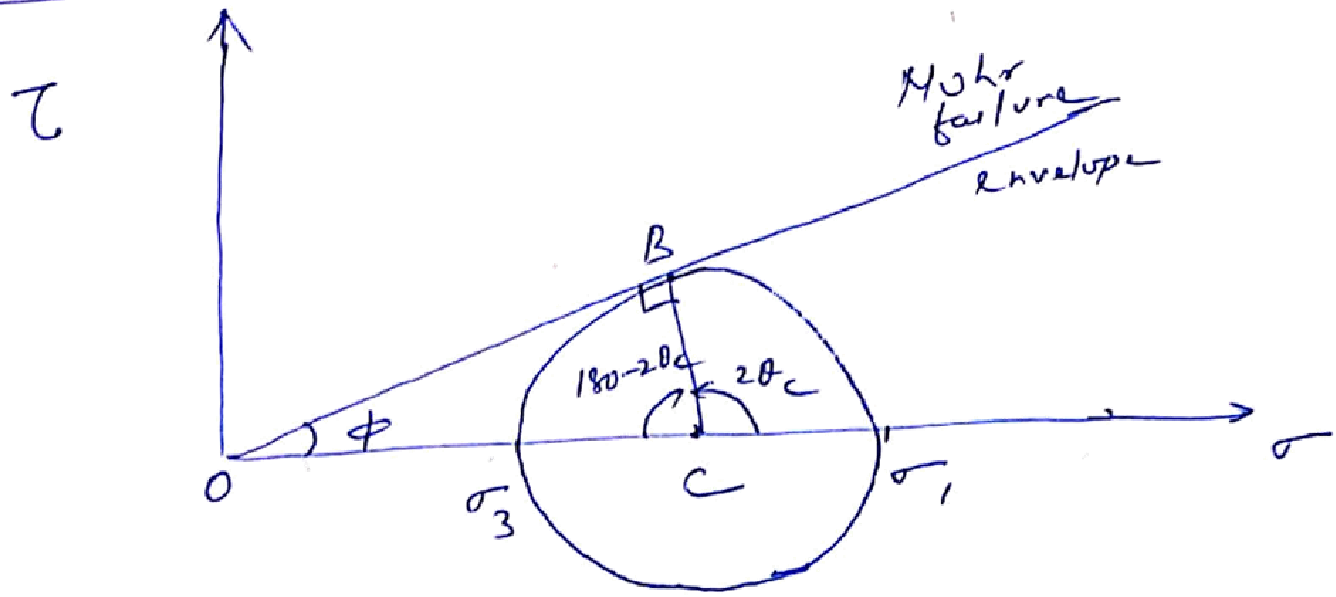
$$\tan\beta = \frac{\tau}{\sigma_n}$$

Where $\beta \rightarrow$ Angle of obliquity.

The shear failure will occur on that plane in which resultant stress (σ_R) is most inclined with the normal of that plane, such plane is called failure plane / Critical plane.

→ If failure plane A–A becomes Critical plane then angle θ is called Critical angle (θ_c)

Mohr – Circle for sand

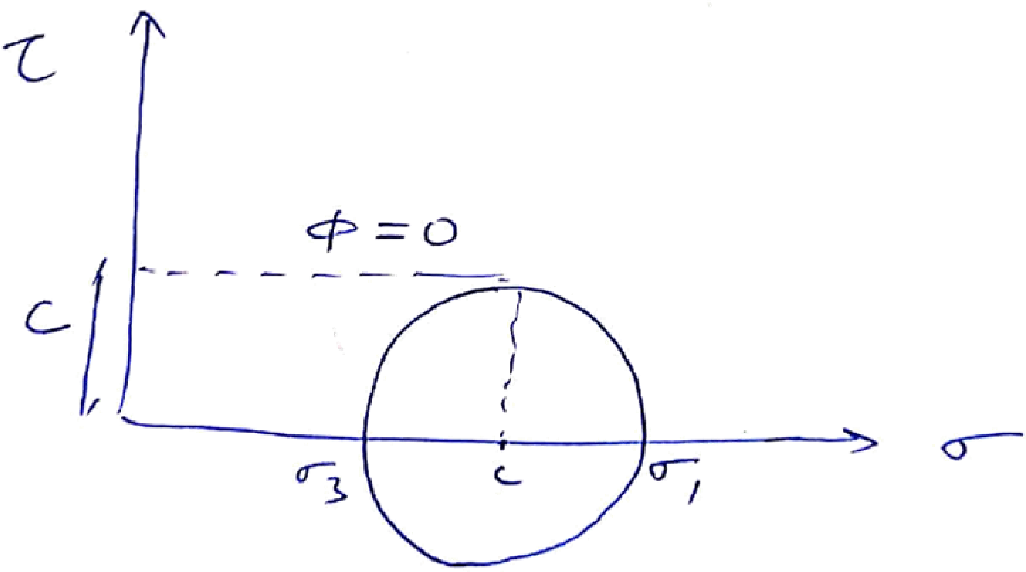


ΔOBC

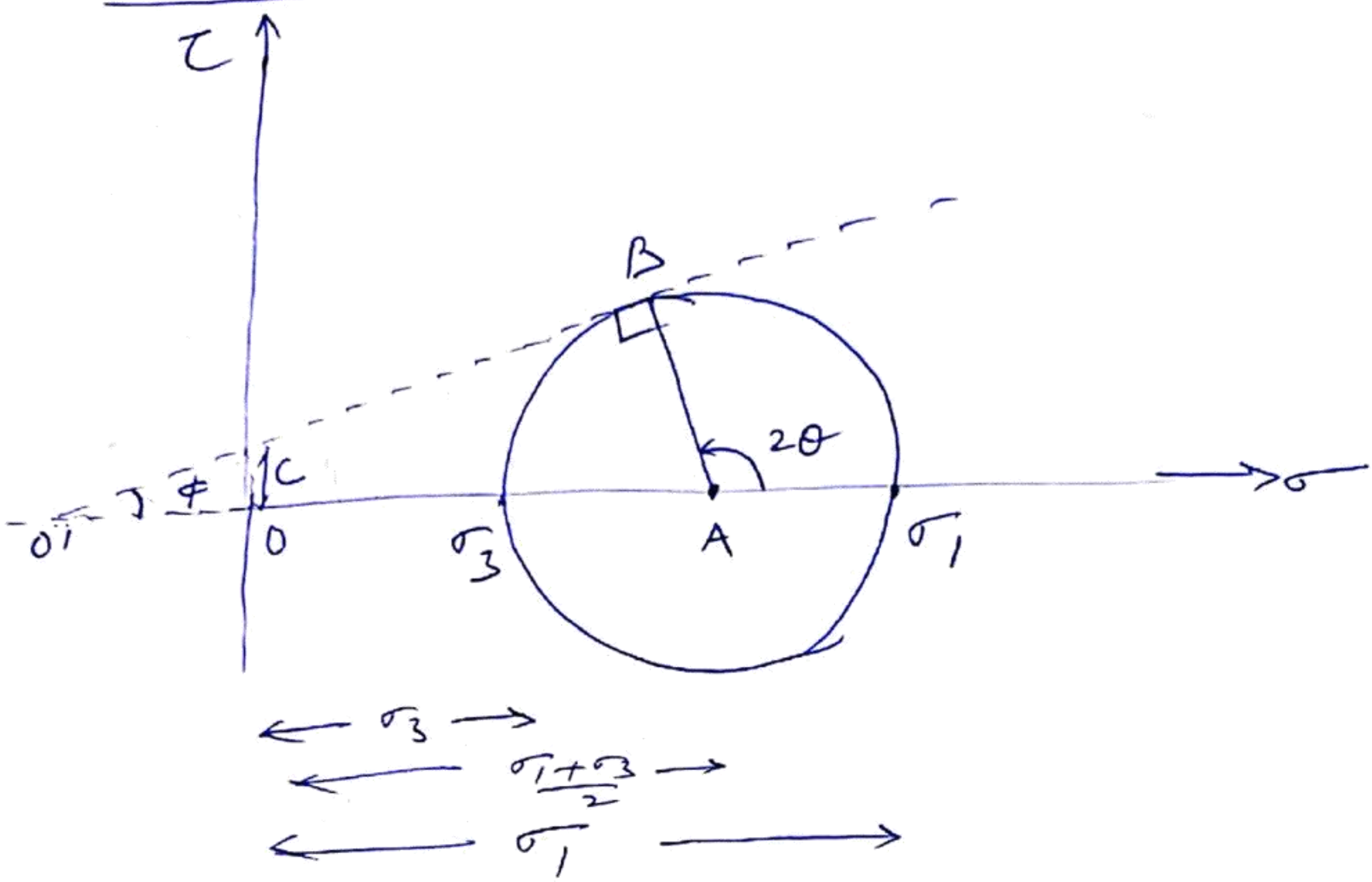
$$\phi + 90 + 180 - 2\theta_c = 180$$

$$\boxed{\theta_c = 45 + \frac{\theta}{2}}$$

Mohr – Circle clay



⇒ Mohr Circle for Silt



In $\Delta\ O'AB$

$$\sin\phi = \frac{AB}{O'A} = \frac{AB}{O'O + OA}$$

$$\sin\phi = \frac{\left(\frac{\sigma_1 - \sigma_3}{2}\right)}{c\cot\phi + \left(\frac{\sigma_1 + \sigma_3}{2}\right)}$$

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = c\cos\phi + \left(\frac{\sigma_1 + \sigma_3}{2}\right)\sin\phi$$

$$\frac{\sigma_1}{2} - \frac{\sigma_1}{2}\sin\phi = c\cos\phi + \frac{\sigma_3}{2} + \frac{\sigma_3}{2}\sin\phi$$

$$\frac{\sigma_1}{2}(1 - \sin\phi) = \frac{\sigma_3}{2}(1 + \sin\phi) + c\cos\phi$$

$$\sigma_1(1-\sin\phi) = \sigma_3(1+\sin\phi) + 2c\cos\phi$$

$$\sigma_1 = \sigma_3\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\frac{\cos\phi}{1-\sin\phi}$$

$$\sigma_1 = \sigma_3\left[\frac{1+\sin\phi}{1-\sin\phi}\right] + 2c\left[\frac{\sqrt{(1+\sin\phi)(1-\sin\phi)}}{1-\sin\phi}\right]$$

$$\sigma_1 = \sigma_3\left[\frac{1+\sin\phi}{1-\sin\phi}\right] + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}$$

$$\boxed{\sigma_1 = \sigma_3\tan^2\left(45+\frac{\phi}{2}\right) + 2c\tan\left(45+\frac{\phi}{2}\right)}$$

$$\boxed{\sigma_1 = \sigma_3\tan^2\alpha + 2c\tan\alpha}$$

α = failure angle = $45 + \frac{\phi}{2}$

⇒ **Mohr – Coulomb Theory:**

A/c to Mohr–Coulomb theory shear strength of soil depends on

(i) Cohesion of soil (c)

(ii) Angle of internal friction (ϕ)

(iii) Normal stress on critical plane or failure plane which increases with depth of soil

Let S be the shear strength of soil

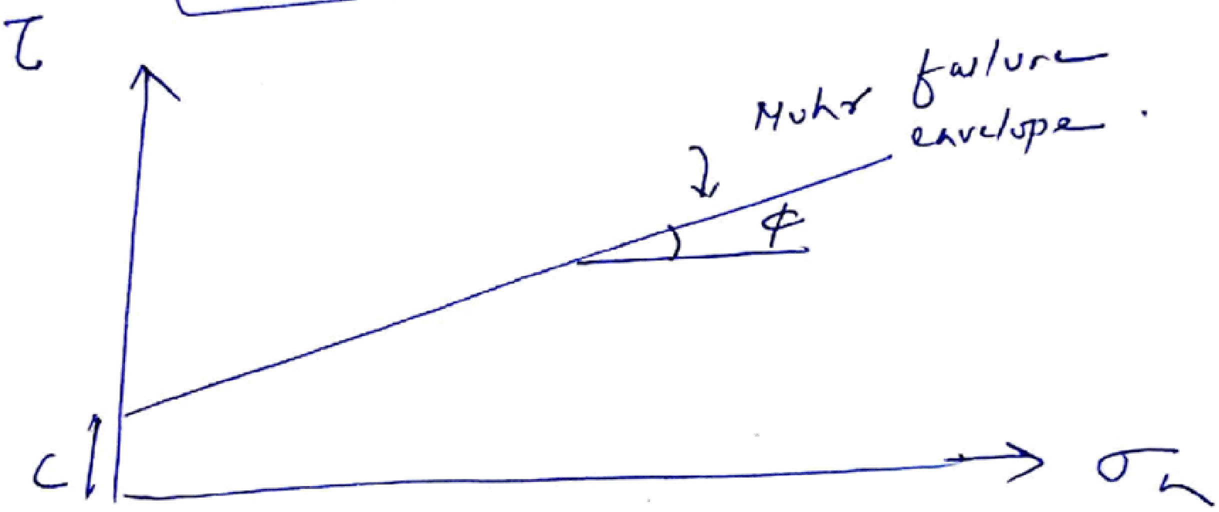
σ_n be the Normal stress

c be cohesion of soil

ϕ → Angle of internal friction

A/c to Mohr-Coulomb theory

$$S = c + \sigma_n \tan\phi$$



→ The above expression is incorrect for the condition where the water table is present. Hence the above theory is modified & improved by Terzaghi which is called Modified Mohr Coulomb theory.

A/c to Modified Mohr-Coulomb

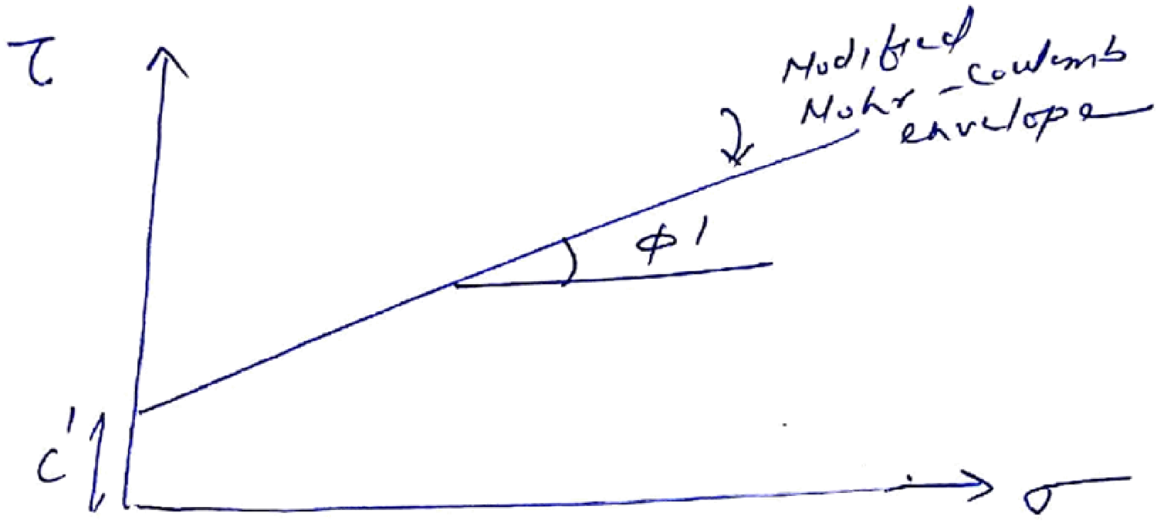
$$S = c' + \sigma_n' \tan\phi'$$

c' → Effective cohesion

σ_n' = Effect stress on the Critical plane

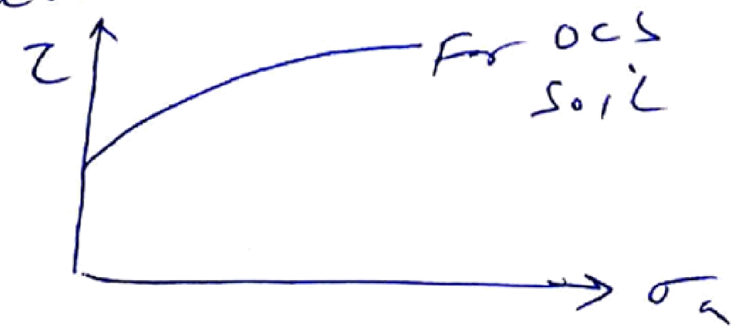
$$\sigma_n' = \sigma_n - U$$

$\phi' \rightarrow$ Effective angle of internal friction.



⇒ Limitations of Mohr-Coulomb theory:

(1) Mohr-Coulomb theory is approximated to a straight line which is found invalid for over consolidated soil. In practical results shows that Mohr-failure envelope in OC soil is found to be slightly curve



(11) The ~~any~~ analysis is 2-D, σ_1 & σ_3 are considered whereas actual stress condition in soil are 3-D in which $\sigma_2 = \sigma_3$.

HAPPY LEARNING