# Department of Civil Engg

Subject: Mechanics of Solid-II

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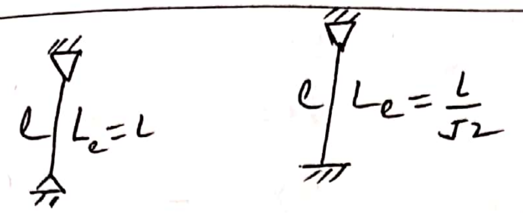
## MID TERM SOLUTION

**Q-1** (i) (b)




$C_1$ $C_2$

$P_{cr_{C_1}} = \frac{\pi^2 EI}{L^2}$ $P_{cr} = \frac{2\pi^2 EI}{L^2}$

Ratio = 1/2

(ii) (d)

1. $\{\sigma, 0\}$ , $\tau_{max} = \sigma/2$
2. $\{\sigma, \sigma\}$ , $\tau_{max} = 0$
3. $(\sigma, -\sigma)$ $\tau_{max} = \sigma$
4. $\{\sigma, \frac{\sigma}{2}\}$ $\tau_{max} = \sigma/4$

2 - 4 - 1 - 3

(iii) (a)

| A | B | C | D |
|---|---|---|---|
| 2 | 3 | 4 | 1 |

(iv) (a)

**Q-2.**

$$\begin{bmatrix} 15 & 8 & -6 \\ 8 & -12 & 5 \\ -6 & 5 & 8 \end{bmatrix} \text{MPa}$$

$I_1 = 15 - 12 + 8 = 11$

$I_2 = -180 - 96 + 120 - 64 - 36 - 25 = -281$

$I_3 = -1440 - 375 + 432 - 512 - 480 = -2375$

$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

$\sigma^3 - 11\sigma^2 - 281\sigma + 2375 = 0$

$\sigma_1 = 19.19$ MPa

$\sigma_2 = 10.27$ MPa , $\sigma_3 = -10.02$ MPa.

Q-3.

$\sigma_x = 100\ N/mm^2$, $\sigma_y = 80\ N/mm^2$

$\tau_{xy} = 50\ N/mm^2$, $\sigma_y = 200\ N/mm^2$, $\nu = 0.30$

(i) A/c to Max^m Principal stress theory:

$$\sigma_1 \leq \frac{\sigma_y}{FOS}$$

$$FOS = \frac{\sigma_y}{\sigma_1} = \frac{200}{\sigma_1}.$$

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= 90 \pm \sqrt{10^2 + 50^2}$$

$$= 90 \pm 50.99$$

$\sigma_1 = 140.99\ N/mm^2$, $\sigma_2 = 39.01\ N/mm^2$

$$FOS = \frac{\sigma_y}{\sigma_1} = \frac{200}{140.99} = \boxed{1.42}.$$

(ii) A/c to Max^m shear stress theory

$$\tau_{max} \leq \frac{\sigma_y}{2(FOS)}$$

$$FOS = \frac{\sigma_y}{2 \times \tau_{max}} = \frac{200}{2 \times \frac{(140.99 - 39.01)}{2}}$$

$$\boxed{FOS = 1.96}$$

Q-4.

$\epsilon_0 = 600 \times 10^{-6}$, $\epsilon_{45} = 500 \times 10^{-6}$

$\epsilon_{90} = 200 \times 10^{-6}$

$$\epsilon_{45} = \epsilon_0 \cos^2\theta + \epsilon_{90} \sin^2\theta + \phi_{xy} \sin\theta . \cos\theta$$

$$500 \times 10^{-6} = 600 \times 10^{-6} \cos^2 45 + (200 \times 10^{-6}) \sin^2 45 + \phi_{xy} \sin 45 . \cos 45$$

$$500\times10^{-6} = 300\times10^{-6} + 100\times10^{-6} + \frac{\phi_{xy}}{2}$$

$$\frac{\phi_{xy}}{2} = 100\times10^{-6}$$

$$\phi_{xy} = 200\times10^{-6}$$

$$\epsilon_{p_1}/\epsilon_{p_2} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= \frac{\epsilon_0+\epsilon_{90}}{2} \pm \sqrt{\left(\frac{\epsilon_{90}-\epsilon_0}{2}\right)^2 + \left(\frac{\phi_{xy}}{2}\right)^2}$$

$$= (400\times10^{-6}) \pm \sqrt{(200\times10^{-6})^2 + (100\times10^{-6})^2}$$

$$= (400\times10^{-6}) \pm (223.6\times10^{-6})$$

$$\boxed{\begin{aligned}\epsilon_{p_1} &= 623\times10^{-6}\\ \epsilon_{p_2} &= 176.4\times10^{-6}\end{aligned}}$$

**Q-5.**

$$\sigma_{xx} = 15, \quad \sigma_{yy} = \sigma_{zz} = 8\ \text{MPa}$$

$$\tau_{xy} = \tau_{yz} = \tau_{xz} = 4\ \text{MPa}$$

$$\sigma_n = \frac{1}{3}(15+8+8) + \frac{2}{3}(6+4+4)$$

$$\boxed{\sigma_n = 19.67\ \text{MPa}}$$

$$\sigma_n^2 + \tau_{ns}^2 = \sigma_r^2$$

$$\tau_{ns} = \sqrt{\sigma_r^2 - \sigma_n^2}$$

$$\sigma_r = \sqrt{\sigma_{rx}^2 + \sigma_{ry}^2 + \sigma_{rz}^2}$$

$$\sigma_{rx} = \sigma_{xx}\, a_{nx} + \tau_{xy}\cdot a_{ny} + \tau_{xz}\, a_{nz}$$

$$= \frac{1}{\sqrt{3}}(15+6+4) = \frac{25}{\sqrt{3}}\ \text{MPa}$$

$$\sigma_{ry} = \frac{1}{\sqrt{3}}(6+8+4) = \frac{18}{\sqrt{3}}\ \text{MPa}$$

$$\sigma_{rz} = \frac{1}{\sqrt{3}}(4+4+8) = \frac{16}{\sqrt{3}}\ \text{MPa}$$

$$\sigma_r = \sqrt{\frac{1}{3}(625+324+256)} = 20.04 \text{ MPa}$$

$$\tau_{ns} = \sqrt{(20.04)^2 - (19.67)^2} = 3.83 \text{ MPa}$$

Q-7. $\epsilon_{xx} = 200$, $\epsilon_{yy} = 100$, $\epsilon_{zz} = 50$ $\mu$ strain

$\phi_{xy} = \phi_{yz} = \phi_{xz} = 40\ \mu$ strain

Normal strain $= \frac{1}{3}(200+100+50) + \frac{1}{3}(40\times3)$

$= 156.67$

$\epsilon_{rx} = \frac{1}{\sqrt{3}}(200+20+20) = \frac{240}{\sqrt{3}}$

$\epsilon_{ry} = \frac{1}{\sqrt{3}}(20+100+20) = \frac{140}{\sqrt{3}}$

$\epsilon_{rz} = \frac{1}{\sqrt{3}}(20+20+50) = \frac{90}{\sqrt{3}}$

Resultant strain ($\epsilon_r$) $= \sqrt{\frac{1}{3}(240^2+140^2+90^2)}$

$= 168.62$

Shearing strain ($\phi_{ns}$) $= 2\sqrt{(168.62)^2 - (156.67)^2}$

$= 124.72$

— END OF THE SOLUTION —